ON THE 107-th, 108-th AND 109-th SMARANDACHE'S PROBLEMS

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Here we shall discuss three definitions regarded by Smarandache as paradoxical.

The analysis of the three definitions is of course a trivial task. Our only motivation for producing it was the desire of making clear the rather imprecise treatment of these paradoxes in [1].

Definition 1. n is called a paradoxist Smarandache number iff n does not belong to any of the Smarandache number sequences.

Let us denote the sequence of paradoxist Smarandache numbers by SP.

Definition 2. n is called non-Smarandache number iff n is neither a Smarandache paradoxist number nor a member of any of the Smarandache defined number sequences.

Let us denote the sequence of non-Smarandache numbers by NS.

We propose two accounts of Definition 1; in both, the apparent paradoxicality is eliminated.

Account 1. If the scope of the definition includes itself, i. e., if a number is called paradoxist iff it does not belong to any Smarandache sequence including SP, then SP is empty.

Proof: Assume SP is not empty and let $p \in$ SP. Then, by the definition, p does not belong to any Smarandache sequence (including SP) and therefore $p \notin$ SP, which is a contradiction with the assumption that $p \in$ SP. Therefore, the contrary holds – that SP is empty.

In other words, SP is not paradoxical by nature, but just an empty sequence.

In this case, NS is equal to

$$N - \bigcup_j S_j$$

where $S_1 \dots S_n$ are all the rest of Smarandache sequences. This is proved by a simple check of Definition 2.

Account 2. If the scope of the definition excludes itself, i. e., if a number is called paradoxist iff it does not belong to any Smarandache sequence except SP, then SP is equal to

$$N - \bigcup_j S_j$$

where $S_1 \dots S_n$ are all the rest of Smarandache sequences.

Let us assume that SP is not empty and let $p \in$ SP. As the definition of SP excludes the SP itself, p does not belong to any Smarandache sequence except SP, and therefore there is no contradiction with the assumption that $p \in$ SP. From the definition it follows that p belongs to the set

$$N - \bigcup_j S_j$$

where $S_1 \dots S_n$ are all of the Smarandache sequences except SP. On the other hand, if SP is empty, then every natural number belongs to some Smarandache sequence other than SP. Since there are no members of SP, and the apparent paradox stemmed from the assumption that some number belongs to SP, no paradox arises in this case either.

Again, there is no paradoxicality here. We cannot, however, make statements about the members of SP in the latter case – it may be empty or not.

In this case, NS is empty. Proof: By definition 2, NS equals

$$N - \bigcup_j S_j - SP$$

and by the above, SP is

$$N - \bigcup_j S_j$$

where S_j are as above. Therefore NS is empty.

Finally, let us consider "the paradox of Smarandache numbers": Any number is a Smarandache number, the non-Smarandache number too.

On both accounts this is true. Therefore it is a mere play of words - it is a matter of choice of the name 'non-Smarandache number' that causes the apparent paradox.

REFERENCE:

[1] Dumitrescu C., V. Seleacu, Some Notions and Questions in Number Theory, Erhus Univ. Press, Glendale, 1994.