ON THE 46-th SMARANDACHE'S PROBLEM

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The 46-th problem from [1] is the following:

Smarandache's prime additive complements:

$$1, 0, 0, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 5, 4, 3, 2, 1, 0, 1, 0,$$

$$5, 4, 3, 2, 1, 0, 3, 2, 1, 0, 5, 4, 3, 2, 1, 0, \ldots$$

(For each n to find the smallest k such that $n + k$ is prime.)

Obviously, the members of the above sequence are differences between first prime number bigger or equal to the current natural number n and the same n. It is well known that the number of primes smaller or equal to n is $\pi(n)$. Therefore, the prime number smaller or equal to n is $p_{\pi(n)}$. Hence, the prime number bigger or equal to n is the next prime number, i.e., $p_{\pi(n)+1}$. Finally, the n-th member of the above sequence will be equal to

$$\begin{cases} p_{\pi(n)+1} - n, & \text{if } n \text{ is not a prime number} \\ 0, & \text{otherwise} \end{cases}$$

We shall note that in [2] the author gives the following new formula p_n for every natural number n:

$$p_n = \sum_{i=0}^{C(n)} sg(n - \pi(i)),$$

where $C(n) = [\frac{n^2 + 3n + 4}{4}]$ (for $C(n)$ see [3]) and

$$sg(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ 1, & \text{if } x > 0 \end{cases},$$

REFERENCES:

[1] C. Dumitrescu, V. Seleacu, Some Sotions and Questions in Number Theory, Erhus Univ. Press, Glendale, 1994.

[5] Atanassov, K. A new formula for the n-th prime number. Comptes Rendus de l'Academie Bulgare des Sciences, Vol. 54, No. 7, 5-6.

[3] Mitrinović, D., M. Popadić. Inequalities in Number Theory. Niś, Univ. of Niś, 1978.