On the 57-th Smarandache's problem *

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Abstract

For any positive integer n, let r be the positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $xy = z$. In this paper, we use the elementary methods to give a sharp lower bound estimate for r.

§1. Introduction

For any positive integer n, let r be a positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $xy = z$. In [1], Schur asks us to find the maximum r. About this problem, it appears that no one had studied it yet, at least, we have not seen such a paper before. The problem is interesting because it can help us to study some important partition problem. In this paper, we use the elementary methods to study Schur's problem and give a sharp lower bound estimate for r. That is, we shall prove the following:

Theorem *For sufficiently large integer n, let r be a positive integer such that: the set $\{1, 2, \cdots, r\}$ can be partitioned into n classes such that no class contains integers x, y, z with $xy = z$. For any number $\varepsilon > 0$, We have*

$$r \geq n^{2(1-\varepsilon)(n-1)}.$$

Whether the upper bound of r is $n^{2(n-1)}$, or there exists another sharper lower bound estimate for r, is an interesting problem.

keywords: Smarandache's problem; Partition; Lower bound.
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§2. Proof of the Theorem

In this section, we complete the proof of the Theorem.

Let $r = \left[n^{2(1-\varepsilon)(n-1)}\right]$ and partition the set $\{1, 2, \cdots, \left[n^{2(1-\varepsilon)(n-1)}\right]\}$ into n classes as follows:

$$\text{Class 1:} \quad 1, \quad \left[n^{(1-\varepsilon)(n-1)}\right], \quad \left[n^{(1-\varepsilon)(n-1)}+1\right], \quad \cdots, \quad \left[n^{2(1-\varepsilon)(n-1)}\right].$$

$$\text{Class 2:} \quad 2, \quad n+1, \quad n+2, \quad \cdots, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots 3}\right].$$

$$\text{Class 3:} \quad 3, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots 3}\right]+1, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots 3}\right]+2, \quad \cdots, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots 4}\right].$$

$$\vdots$$

$$\text{Class k:} \quad k, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots k}\right]+1, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots k}\right]+2, \quad \cdots, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots (k+1)}\right].$$

$$\vdots$$

$$\text{Class n:} \quad n, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n}\right]+1, \quad \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n}\right]+2, \quad \cdots, \quad \left[n^{(1-\varepsilon)(n-1)}-1\right].$$

where $[y]$ denotes the integer part of y.

It is obvious that Class k contains no integers x, y, z with $xy = z$ for $k = 1, 3, 4, \cdots, n$. In fact for any integers $x, y, z \in$ Class k, $k = 3, 4, \cdots, n$, we have

$$xy \geq k \times \left(\left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots k}\right]+1\right) > \left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots (k+1)}\right] \geq z.$$

On the other hand, $\left[\frac{(n^{(1-\varepsilon)(n-1)}-1)}{n(n-1)\cdots 3}\right]$ tends to zero when $n \dashrightarrow \infty$, so for sufficiently large integer n, Class 2 has only one integer 2.

This completes the proof of the Theorem.

References

[1] F. Smarandache, Only problems, not Solutions, Xiquan Publ. House, Chicago, 1993, 48.

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[3] "Smarandache Sequences" at http://www.gallup.unm.edu/ ˜smarandache/snaqint2.txt.

[4] "Smarandache Sequences" at http://www.gallup.unm.edu/ ˜smarandache/snaqint3.txt.