ON THE BALU NUMBERS

Maohua Le

Abstract . In this paper we prove that there are only finitely many Balu numbers.

Key words . Smarandache factor partition , number of divisors , Balu number , finiteness .

1.Introductio

For any positive integer n, let $d(n)$ and $f(n)$ be the number of distinct divisors and the Smarandache factor partitions of n respectively. If n is the least positive integer satisfying

(1) $$d(n)=f(n)=r$$

for some fixed positive integers r, then n is called a Balu number. For example , n=1,16 36 are Balu numbers. In [4], Murthy proposed the following conjecture.

Conjecture. There are finitely many Balu numbers.

In this paper we completely solve the mentioned question. We prove the following result.

Theoem . There are finitely many Balu numbers.

2.Preliminaries

For any positive integer n with n>1, let

(2) $$n=p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$$

be the factorization of n.

Lemma 1 ([1, Theorem 273]) . $d(n)=(a_1+1)(a_2+1)\ldots(a_k+1)$.

Lemma 2. Let a, p be positive integers with $p>1$, and let

(3) $$b=\left[\frac{1}{2}\sqrt{1+8a}-\frac{1}{2}\right].$$

Then p^a can be written as a product of b distinct positive integers

(4) $$p^a=p.p^2\ldots p^{b-1}p^{a-b(b-1)/2}.$$

Proof. We see from (3) that $a \geqslant 1+2+\ldots+(b-1)+b$. Thus, the lemma is true.

Lemma 3. For any positive integer m, let $Y(m)$ be the m-th Bell number. Then we have

(5) $$f(n) \geqslant Y(c),$$

where

(6) $$c=b_1+b_2\cdots+b_k$$

and

(7) $$b_i=\left[\frac{1}{2}\sqrt{1+8a_i}-\frac{1}{2}\right],\ i=1,2,\cdots,k.$$

Proof. Since $p_1, p_2, \ldots p_k$ are distinct primes in the factorization (2) of n, by Lemma 2, we see from (6) and (7) that n can be written as a product of c distinct postitive integers

(8) $$n=\prod_{i=1}^{k}\left(p_i^{a_i-b_i(b_i-1)/2}\prod_{j=1}^{b_i-1}p_i^{j}\right).$$

Therefore, by (6) and (8), we get

(9) $$f(n) \geqslant F(1\#c),$$

where $F(1\#c)$ is the number of Smarandache factor partitions of a product of c distinct primes. Further, by [2,Theorem], we have

(10) $$F(1\#c)=Y(c).$$

Thus, by (9) and (10), we obtain (5). The lemma is proved.

Lemma 4 ([3]). $\log Y(m) \sim m \log m$.

3.Proof of Theorem

We now suppose that there exist infinitely many Balu numbers. Let n be a Balu number, and let (2) be the factorization of n. Further, let

$$a=a_1+a_2+\dots+a_k. \tag{11}$$

Clear, if n is enough large, then a tends to infinite. Morever, since

$$b_i \geqslant \sqrt{a} \quad i=1,2,\cdots,k, \tag{12}$$

by (7), we see from (6) that c tends to infinite too. Therefore, by Lemmas 1, 3 and 4, we get from (1), (2), (6) and (12) that

$$1=\frac{\log d(n)}{\log f(n)} \leqslant \frac{\sum\limits_{i=1}^{k} \log(a_i+1)}{\left(\sum\limits_{i=1}^{k} \sqrt{a_i}\right)\left(\log \sum\limits_{i=1}^{k}\sqrt{a_i}\right)} \leqslant \frac{\sum\limits_{i=1}^{k} \log(a_i+1)}{k\sum\limits_{i=1}^{k} \sqrt{a_i}} < 1, \tag{13}$$

a contradiction. Thus, there are finitely many Balu numbers. The theorem is provde.

References

[1] G. H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Oxford University Press, Oxford, 1937.

[2] M.-H. Le, Two conjectures concerning extents of Smarandache factor partitions, Smarandache Notions J., **12(2001).**

[3] L. Moser and M. Wyman, An asymptotic formula for the Bell numbers, Trans. Roy Sci. Canada 49 (1955), 49-54.

[4] A. Murthy, Open problems and conjectures on the factor / reciprocal partition theory, Smarandache Notions J. 11(2000), 308-311.

Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA