ON THE DIOPHANTINE EQUATION S(n) = n

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Abstract. Let $S(n)$ denote the Smarandache function of n. In this paper we prove that $S(n) = n$ if and only if $n = 1, 4$ or p, where p is a prime.

Let N be the set of all positive integers. For any positive integer n, let $S(n)$ denote the Smarandache function of n (see[1]). It is an obvious fact that $S(n) \leq n$. In this paper we consider the diophantine equation

$$(1) \qquad S(n) = n, \; n \in N.$$

We prove a general result as follows:

Theorem. The equation (1) has only the solutions $n = 1,4$ or p, where p is a prime.

Proof. If $n = 1,4$ or p, then (1) holds. Let n be an another solution of (1). Then n must be a composite integer with $n > 4$. Since n is a composite integer, we have $n = uv$, where u,v are integers satisfying $u \geq v \geq 2$. If $u \neq v$, then we get $n \mid u!$. It implies that $S(n) \leq u = n / v < n$, a contradiction.

If $u = v$, then we have $n = u^2$ and $n \mid (2u)!$
It implies that $S(n) \leq 2u$. Since $n > 4$, we get $u > 2$ and $S(n) \leq 2u < u^2 = n$, a contradiction. Thus, (1) has only the solution $n = 1,4$ or p. The theorem is proved.

Reference

1. F Smarandache, A function in the number theory, Smarandache function J. 1 (1990), No.1, 3 - 17.