ON THE DIVISORS OF SMARANDACHE UNARY SEQUENCE

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ABSTRACT: Smarandache Unary Sequence is defined as follows:

u(n) = 1111 . . ., p_n digits of "1" , where p_n is the n^{th} prime.

11, 111, 11111, 1111111 . . .
Are there an infinite number of primes in this sequence ? It is still an unsolved problem. The following property of a divisor of u(n) is established.
If 'd' is a divisor of u(n) then $d \equiv 1 \pmod{p_n}$. , for all n > 3 ---(1),

DESCRIPTION: Let $l(m) = 1111$,. . . m times $= (10^m - 1)/9$

Then $u(n) = l(p_n)$.
Following proposition will be applied to establish (1).
Proposition : $l(p-1) \equiv 0 \pmod p$.-------(2)
PROOF: 9 divides $10^{p-1} - 1$. From Fermat's little theorem if $p \geq 7$ is a prime then p divides $(10^{p-1} - 1)/9$
as (p , 9) = (p , 10) = 1. Hence p divides l(p-1)

Coming back to the main proposition , let 'd' be a divisor of u(n).

Let d =. $p^a q^b r^c$. . ,where p , q, r, are prime factors of d .

p divides| 'd' $\Rightarrow$ p divides u(n) also p divides l(p-1) from proposition (2). in other words

p divides $(10^{p-1} - 1)/9$ and p divides $(10^p - 1)/9$

p divides $(10^{A(p-1)} - 1)/9$ and p divides $(^{10B.p} - 1)/9$

p divides $(10^{(A(p-1) - B.p})/9$

p divides $10^{B.p}\{(10^{A(p-1) - B.p} - 1)/9\}$.

p divides $(10^{A(p-1) - B.p} - 1)/9$. ---------(3)

There exist A and B such that

$A(p - 1) - B.p_n = (p - 1, p_n)$. As p_n is a prime there are two possibilities :

(i). $(p - 1, p_n) = 1$ or (ii). $(p - 1, p_n) = p_n$.

In the first case , from (3) we get p divides (10 - 1)/9 or p

divides 1, which is absurd as $p > 1$. hence $(p - 1, p_n) = p_n$;
or p_n divides $p - 1$

$$p \equiv 1 \pmod{p_n}$$

$$\Rightarrow \quad p^a \equiv 1 \pmod{p_n}$$

on similar lines

$$q^b \equiv 1 \pmod{p_n}$$

hence $d = p^a q^b r^c \ldots \equiv 1 \pmod{p_n}$

This completes the proof.

COROLLARY : For any prime p there exists at least one prime q such that
$q \equiv 1 \pmod{p}$

Proof: As $u(n) \equiv 1 \pmod{p_n}$, and also every divisor of u(n) is

$\equiv 1 \pmod{p_n}$, the corollary stands proved. Also clearly such a 'q' is greater than p , this gives us a proof of the infinitude of the prime numbers as a by product.

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