ON THE EQUATION $S(mn)=m^kS(n)$

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Abstract. In this paper we prove that the equation $S(mn)=m^kS(n)$ has only the positive integer solution $(m,n,k)=(2,2,1)$ with $m>1$ and $n>1$.

Key words Smarandache function, equation, positve integer solution .

For any positive integer a,let $S(a)$ be the Smarandache function.Muller [2,Problem 21] proposed a problem concerning the integer solutions (m,n,k) of the equation

(1) $$S(mn)=m^kS(n), \quad m>1, n>1.$$

In this paper we determine all solutions of (1) as follows.

Theorem. The equation (1) has only the solution $(m,n,k)=(2,2,1)$.

Proof. By [1,Theorem],we have

(2) $$S(mn) \leqslant S(m)+S(n).$$

Hence,if (m,n,k) is a solution of (1),then from (2)we obtain

(3) $$m^kS(n) \leqslant S(m)+S(n).$$

By (3),we get

(4) $$m^k \leqslant \frac{S(m)}{S(n)}+1 .$$

Since $S(m) \leqslant m$,we see from (4)that

(5) $$m^k \leqslant \frac{m}{S(n)}+1 .$$

If $n>2$,then $S(n) \geqslant 3$ and

(6) $$m \leqslant mk \leqslant \frac{m}{3}+1,$$

by(5). However,we get from (6) that $m \leqslant 1/2$,a contradiction. So we have $n=2$. Then, we get $S(n)=2$ and

$$(7) \qquad m^k \leqslant \frac{m}{2} + 1$$

by (5).

If $m>2$, then $m/2>1$, and

$$(8) \qquad m \leqslant m^k < \frac{m}{2} + \frac{m}{2} = m$$

by (7). This is a contradiction. Therefore, we get $m=2$ and

$$(9) \qquad 2^k \leqslant 1+1=2,$$

by (7). Thus, we see from (9) that (1) has only the solution $(m,n,k)=(2,2,1)$. The theorem is proved.

References

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