ON THE SMARANDACHE N-ARY SIEVE

Maohua Le

Department of Mathematics, Zhanjiang Normal College
Zhanjiang, Guangdong, P.R.China.

Abstract. Let n be a positive integer with $n > 1$. In this paper we prove that the remaining sequence of Smarandache n-ary sieve contains infinitely many composite numbers.

Let n be a positive integer with $n > 1$. Let S_n denote the sequence of Smarandache n-ary sieve (see [1, Notions 29-31]). For example:

$S_2=\{1,3,5,9,11,13,17,21,25,27, \ldots\}$,

$S_3=\{1,2,4,5,7,8,10,11,14,16,17,19,20, \ldots\}$

In [1], Dumitrescu and Seleacu conjectured that S_n contains infinitely many composite numbers. In this paper we verify the above conjecture as follows:

Theorem. For any positive integer n with $n>1$, S_n contains infinitely many composite numbers.

Proof. By the definition of Smarandache n -ary sieve (see [1, Notions 29-31]), the sequence S_n contains the numbers $n^k + 1$ for any positive integer k. If k is an odd integer with $k > 1$, then we have

$$(1) \qquad n^k + 1 = (n+1)(n^{k-1} - n^{k-2} + \dots + 1).$$

We see from (1) that $(n+1) \mid (n^k + 1)$ and $n^k + 1$ is a composite number. Notice that there exist infinitely many odd integers k with $k > 1$. Thus, S_n contains infinitely many composite numbers $n^k + 1$. The theorem is proved.

References.

1. Dumitrescu and V. Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994.