

ON THE SYMMETRIC SEQUENCE AND ITS SOME PROPERTIES*

ZHANG WENPENG

Research Center for Basic Science, Xi'an Jiaotong University
Xi'an, Shaanxi, P.R.China

ABSTRACT. The main purpose of this paper is to prove that there is only one prime among the symmetric sequence. This solved the problem 17 of Professor F.Smarandache in [1]. *for $n \geq 9$ (partially)*

1. INTRODUCTION

For any positive integer n , we define the symmetric sequence $\{S(n)\}$ as follows: $S(1) = 1$, $S(2) = 11$, $S(3) = 121$, $S(4) = 1221$, $S(5) = 12321$, $S(6) = 123321$, $S(7) = 1234321$, $S(8) = 12344321$, \dots . In problem 17 of [1], Professor F.Smarandache asked us to solve such a problem: How many primes are there among these numbers? This problem is interesting, because it can help us to find some new symmetric primes. In this paper, we shall study this problem, and give an exact answer. That is, we shall prove the following conclusion:

Theorem. *For any positive integer $n \geq 2$, we have the decomposition*

$$\begin{aligned} & \{ n \leq 9 \\ 123 \cdots (n-1)nn(n-1) \cdots 321 &= \overbrace{11 \cdots 1}^n \times \overbrace{11 \cdots 1}^{n+1}; \\ 123 \cdots (n-1)n(n-1) \cdots 321 &= \overbrace{11 \cdots 1}^n \times \overbrace{11 \cdots 1}^n. \end{aligned}$$

From this theorem we may immediately deduce the following two corollaries:

Corollary 1. *There is only one prime among the symmetric sequence, That is, $S(2) = 11$. *for $n \leq 9$.**

Corollary 2. *For any positive integer $n \leq 9$, $S(2n-1)$ is a perfect square number. That is,*

$$\begin{aligned} S(2n-1) &= 123 \cdots (n-1)n(n-1) \cdots 321 \\ &= \overbrace{11 \cdots 1}^n \times \overbrace{11 \cdots 1}^n. \end{aligned}$$

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2. PROOF OF THE THEOREM

In this section, we complete the proof of the theorems. First we let

$$S_1 = \{11, 1221, 123321, \dots, 123 \cdots (n-1)nn(n-1) \cdots 321, \dots, \}, \text{ for } n \leq 9,$$

and

$$S_2 = \{1.121, 12321, \dots, 123 \cdots (n-1)n(n-1) \cdots 321, \dots, \}, \text{ for } n \leq 9.$$

Then it is clear that

$$\{S(n)\} = S_1 \cup S_2.$$

For any positive integer $m \in \{S(n)\}$, we have $m \in S_1$ or $m \in S_2$. If $m \in S_1$, then there exists a positive integer n such that $m = 123 \cdots (n-1)nn(n-1) \cdots 321$. So that

$$\begin{aligned} m &= 10^{2n-1} + 2 \times 10^{2n-2} + \cdots + n \times 10^n \\ &\quad + n \times 10^{n-1} + (n-1) \times 10^{n-2} + \cdots + 2 \times 10 + 1 \\ &= [10^{2n-1} + 2 \times 10^{2n-2} + \cdots + n] \times 10^n \\ &\quad + [n \times 10^{n-1} + (n-1) \times 10^{n-2} + \cdots + 2 \times 10 + 1] \\ (1) \quad &\equiv S_{11} + S_{12}. \end{aligned}$$

Now we compute S_{11} and S_{12} in (1) respectively. Note that

$$\begin{aligned} 9S_{11} &= 10S_{11} - S_{11} = 10^{2n} + 2 \times 10^{2n-1} + \cdots + n \times 10^{n+1} \\ &\quad - 10^{2n-1} - 2 \times 10^{2n-2} - \cdots - n \times 10^n \\ &= 10^{2n} + 10^{2n-1} + 10^{2n-2} + \cdots + 10^{n+1} - n \times 10^n \\ &= 10^{n+1} \times \frac{10^n - 1}{9} - n \times 10^n \end{aligned}$$

and

$$\begin{aligned} 9S_{12} &= 10S_{12} - S_{12} = n \times 10^n + (n-1) \times 10^{n-1} + \cdots + 2 \times 10^2 + 10 \\ &\quad - n \times 10^{n-1} - (n-1) \times 10^{n-2} - \cdots - 2 \times 10 - 1 \\ &= n \times 10^n - 10^{n-1} - 10^{n-2} - \cdots - 10 - 1 \\ &= n \times 10^n - \frac{10^n - 1}{9}. \end{aligned}$$

So that we have

$$(2) \quad S_{11} = \frac{1}{81} \times [10^{2n+1} - 9n \times 10^n - 10^{n+1}]$$

and

$$(3) \quad S_{12} = \frac{1}{81} [9n \times 10^n - 10^n + 1].$$

Combining (1), (2) and (3) we have

$$\begin{aligned}
 m &= S_{11} + S_{12} \\
 &= \frac{1}{81} \times [10^{2n+1} - 9n \times 10^n - 10^{n+1}] + \frac{1}{81} [9n \times 10^n - 10^n + 1] \\
 &= \frac{1}{81} (10^{2n+1} - 10^{n+1} - 10^n + 1) \\
 &= \frac{1}{81} (10^n - 1)(10^{n+1} - 1) \\
 (4) \quad &= \underbrace{11 \cdots 1}_n \times \underbrace{11 \cdots 1}_{n+1}.
 \end{aligned}$$

If $m \in S_2$, then there exists a positive integer n such that

$$m = 123 \cdots (n-1)n(n-1) \cdots 321.$$

Similarly, we also have the identity

$$\begin{aligned}
 m &= 10^{2n-2} + 2 \times 10^{2n-3} + \cdots + n \times 10^{n-1} \\
 &\quad + (n-1) \times 10^{n-2} + (n-2) \times 10^{n-3} + \cdots + 2 \times 10 + 1 \\
 &= \frac{1}{81} (10^{2n} - 10^n - 9n \times 10^{n-1}) + \frac{1}{81} (9n \times 10^{n-1} - 10^n + 1) \\
 (5) \quad &= \left[\frac{10^n - 1}{9} \right]^2 = \underbrace{11 \cdots 1}_n \times \underbrace{11 \cdots 1}_n.
 \end{aligned}$$

Now the theorem 1 follows from (4) and (5).

From theorem 1 we know that $S(n)$ is a composite number, if $n \geq 3$, ^(and $n \leq 9$) Note that $S(1) = 1$ and $S(2) = 11$ (a prime), we may immediately deduce the theorem 2. This completes the proof of the theorems.

REFERENCES

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