# ON THE THIRD SMARANDACHE CONJECTURE ABOUT PRIMES

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**Abstract**. In this paper we basically verify the third Smarandache conjecture on prime.

**Key words**. Smarandache third conjecture, prime, gap.

For any positive integre $n$, let $P(n)$ be the $n$-th prime. Let $k$ be a positive integer with $k>1$, and let

$$c(n,k)=(P(n+1))^{1/k}-(P(n))^{1/k}. \tag{1}$$

Smarandache [3] has been conjectured that

$$C(n,k)<\frac{2}{k}. \tag{2}$$

In [2], Russo verified this conjecture for $P(n)<2^{25}$ and $2\leqslant k\leqslant 10$. In this paper we prove a general result as follows.

**Theorem**. If $k>2$ and $n>C$, where $C$ is an effectively computable absolute constant, then the inequality (2) holds.

**Proof**. Since $k>2$, we get from (1) that

$$C(n,k)=\frac{P(n+1)-P(n)}{(P(n+1))^{(k-1)/k}+(P(n+1))^{(k-2)/k}(P(n))^{1/k}+\ldots+(P(n))^{(k-1)/k}}$$

$$<\frac{P(n+1)-P(n)}{k(P(n))^{(k-1)/k}}\leqslant\frac{2}{k}\left(\frac{(P(n+1)-P(n)}{2(P(n)^{2/3}}\right). \tag{3}$$

By the result of [1], we have

$$P(n+1)-P(n))<C(a)(P(n))^{11/20+a}, \tag{4}$$

for any positive number $a$, where $C(a)$ is an effectively computable constant depending on $a$. Put $a=1/20$ . Since $k\geq 3$ and $(k-1)/k\geq 2/3$, we see from (3) and (4) that

$$C(n,k)<\frac{2}{k}\left(\frac{C(1/20)}{2(P(n))^{1/15}}\right) . \tag{5}$$

Since $C(1/20)$ is an effectively computable absolute constant, if $n>C$, then $2(P(n))^{1/15}>C(1/20)$. Thus, by (5), the inequality (2) holds. The theorem is proved.

## References


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