OPEN PROBLEMS AND CONJECTURES ON THE FACTOR /RECIPROCAL PARTITION THEORY:

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(1.1) To derive a formula for SFPs of given length m of $p^a q^a$ for any value of a.

(1.2) To derive a formula for SFPs of

$$N = p_1^2 \; p_2^2 \; p_3^2 \; . \; . \; . \; p_r^2$$

(1.3) To derive a formula for SFPs of given length m of

$$N = p_1^\alpha \; p_2^\alpha \; p_3^\alpha \; . \; . \; . \; p_r^\alpha$$

(1.4) To derive a reduction formula for $p^a q^a$ as a linear combination of $p^{a-r} q^{a-r}$ for r =0 to a-1.

Similar reduction formulae for (1.2) and (1.3) also.

(1.5) In general , in how many ways a number can be expressed as the product of its divisors?

(1.6). Every positive integer can be expressed as the sum of the reciprocal of a finite number of distinct natural numbers. (in infinitely many ways.).

Let us define a function $R_m(n)$ as the minimum number of natural numbers required for such an expression.

(1.7). Every natural number can be expressed as the sum of the reciprocals of a set of natural numbers which are in Arithmetic Progression.

(1.8). Let

$\sum 1/r \leq n \leq \sum 1/(r+1)$

where $\sum 1/r$ stands for the sum of the reciprocals of first r natural numbers and let $S_1 = \sum 1/r$

let $S_2 = S_1 + 1/(r+k_1)$ such that $S_2 + 1/(r+k_1+1) > n \geq S_2$

let $S_3 = S_2 + 1/(r+k_2)$ such that $S_3 + 1/(r+k_2+1) > n \geq S_3$

and so on , then there exists a finite m such that

$S_{m+1} + 1/(r+k_m) = n$

Remarks : The veracity of conjecture (1.6) is deducible from conjecture (1.8) .

(1.9). (a) There are infinitely many disjoint sets of natural numbers sum of whose reciprocals is unity.

(b) Among the sets mentioned in (a) , there are sets which can be organised in an order such that the largest element of any set is smaller than the smallest element of the next set.

DEFINITION: We can define **Smarandache Factor Partition Sequence** as follows : T_n = factor partion of $n = F'(n)$

$T_1 = 1$. $T_8 = 3$. $T_{12} = 4$ etc.

SFPS is given by

1,1,1,2,1,2,1,3,2,2,1,4,1,2,2,5, 1,4,1,4,2,2,1,7,2, . . .,

DEFINITION: Let S be the smallest number such that $F'(S) = n$. We define S a **Vedam Number** and the sequence formed by Vedam numbers as the **Smarandache Vedam Sequence**.

Smarandache Vedam Sequence is given as follows: $T_n = F'(S)$

1, 4, 8, 12, 16, **-?-** , 24 , . . .

Note: There exist no number whose factor partition is equal to 6. hence a question mark at the sixth slot. We define such numbers as **Dull numbers.** The readers can explore the distribution (frequency) and other properties of dull numbers.

DEFINITION: A number n is said to be a **Balu number** if it satisfies the relation $d(n) = F'(n) = r$, and is the smallest such number .

1, 16, 36 are all Balu numbers.

$d(1) = F'(1) = 1$ $d(16) = F'(16) = 5$, $d(36) = F'(36) = 9$.

Each Balu number ≥ 16 , generates a **Balu Class** $C_B(n)$ of numbers having the same canonical form satisfying the equation $d(m) = F'(m)$.e.g. $C_B(16) = \{ x \mid x = p^4$, p is a prime.$\} = \{ 16, 81, 256, \ldots\}$. Similarly $C_B(36) = \{ x \mid x = p^2q^2$, p and q are primes.$\}$

Conjecture

(1.10): There are only finite number of Balu Classes.

In case Conjecture (1.10) is true , to find out the largest **Balu number.**

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