Open Questions For The Smarandache Function

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Let S(n) be the Smarandache function. I propose the following open questions:

1) Solve the following equation in integers:

$$1/S^2(a) = 1/S^2(b) + 1/S^2(c).$$

2) Solve the following equation in integers:

$$S^2(\phi(a)) = S^2(\phi(b)) + S^2(\phi(c)).$$

3) Solve the following equation in integers:

$$S(d(n) + \sigma(n)) = d(S(n)) + \sigma(S(n)).$$

4) Solve the following equation in integers:

$$S(a*d(n) + b*\sigma(n) + c*\phi(n) + d*\psi(n)) = a*d(S(n)) + b*\sigma(S(n)) + c*\phi(S(n)) + d*\psi(S(n)).$$

5) Solve the following equation in integers:

$$S\left(\sum_{k=1}^{n} n^k\right) = \prod_{k=1}^{n} S(k)*\phi(n)$$

6) Solve the following equation in integers:

$$\pm S(1) \pm S(2) \pm \ldots \pm S(n) = \phi((n(n+1))/2).$$

7) Solve the following equation in integers:

$$S(\pm 1^2 \pm 2^2 \pm \ldots \pm n^2) = \pm S^2(1) \pm S^2(2) \pm \ldots \pm S^2(n).$$

8) Solve the following equation in integers:

$S(\pm \mu(1) \pm \mu(2) \pm \ldots \pm \mu(n)) = S(\mu((n(n+1))/2)$.

9) Solve the following equation in integers:

$S(\pm d(1) \pm d(2) \pm \ldots \pm d(n)) = S(d((n(n+1))/2)$.

10) Solve the following equation in integers:

$S(\pm \sigma(1) \pm \sigma(2) \pm \ldots \pm \sigma(n)) = S(\sigma((n(n+1))/2)$.

11) Solve the following equation in integers:

$$\frac{1}{S(1)} + \frac{1}{S(2)} + \ldots + \frac{1}{S(n)} = \frac{n}{S((n(n+1))/2)}.$$

12) Solve the following equation in integers:

$S(1*2) + S(2*3) + \ldots + S(n(n+1)) = S((n(n+1)(n+2))/3)$.

13. Let $\alpha_k(n)$ be the first k digits of n and $\beta_p(n)$ the last p digits of n. Determine all integer 5-tuples (n,m,r,k,p) for which:

$S^2(\alpha_k(n)) = S^2(\alpha_k(m)) + S^2(\alpha_k(r))$

and

$S^2(\beta_p(n)) = S^2(\beta_p(m)) + S^2(\beta_p(r))$.

14) Determine all integer 5-tuples (n,m,r,k,p) for which:

$\alpha_k^2(S(n)) = \alpha_k^2(S(m)) + \alpha_k^2(S(r))$

and

$\beta_k^2(S(n)) = \beta_k^2(S(m)) + \beta_k^2(S(k))$.

15) Determine all integer pairs (n,k) for which:

$\alpha^2_{k+2}(S(n)) = \alpha^2_{k+1}(S(n)) + \alpha_k^2(S(n))$.

16) Determine all integer pairs (n,p) for which:

$\beta^2{}_{p+2}(S(n)) = \beta^2{}_{p+1}(S(n)) + \beta_p{}^2(S(n))$.

17) Find all integer pairs (n,k) such that

$S(\alpha_k(n)) + S(\alpha_k(n+2)) = 2 * S(\alpha_k(n+1))$

18) Find all integer pairs (n,p) such that

$$S(\beta_p(n)) + S(\beta_p(n+2)) = 2 * S(\beta_p(n+1))$$

19) Let p_n be the n-th prime number. Determine all integer triples (n,k,p) for which

$$S(\alpha_k(p_n)) + S(\beta_p(p_n)) = 2 * S(\alpha_{(k+p)/2}(p_n)$$

and

$$S(\alpha_k(p_n)) + S(\beta_p(p_n)) = 2 * S(\beta_{(k+p)/2}(p_n).$$

20) Find all integer pairs (a,b) such that

$$\frac{a*S(b) + b*S(a)}{a+b} = S\left[\frac{a^2+b^2}{a+b}\right]$$

21) Solve the following in integers:

$\pm S(\sigma(1)) \pm S(\sigma(2)) \pm \ldots \pm S(\sigma(n)) = \pm\sigma(\pm S(1) \pm S(2) \pm \ldots \pm S(n))$.

22) Solve the following in integers:

$\alpha_k(n) = S(n)$ and $\beta_p(n) = S(n)$.

23) Solve the following in integers:

$\alpha_k(n!) = S(m)$ and $\beta_p(n!) = S(m)$.