Palindrome Studies
(Part I)

The Palindrome Concept and Its Applications to Prime Numbers

Henry Ibstedt
Glimminge 2036
280 60 Broby
Sweden

Abstract: This article originates from a proposal by M. L. Perez of American Research Press to carry out a study on Smarandache generalized palindromes [1]. The prime numbers were chosen as a first set of numbers to apply the development of ideas and computer programs on. The study begins by exploring regular prime number palindromes. To continue the study it proved useful to introduce a new concept, that of extended palindromes with the property that the union of regular palindromes and extended palindromes form the set of Smarandache generalized palindromes. An interesting observation is proved in the article, namely that the only regular prime number palindrome with an even number of digits is 11.

1. Regular Palindromes

Definition: A positive integer is a palindrome if it reads the same way forwards and backwards.

Using concatenation we can write the definition of a regular palindrome A in the form

$$A=x_1x_2x_3...x_n...x_3x_2x_1 \text{ or } x_1x_2x_3...x_nx_n...x_3x_2x_1$$

where $x_k \varepsilon \{0, 1, 2, \ldots 9\}$ for $k=1, 2, 3, \ldots n$, except $x_1 \neq 0$

Examples and Identification: The digits 1, 2, ..., 9 are trivially palindromes. The only 2-digit palindromes are 11, 22, 33, ... 99.
Of course, palindromes are easy to identify by visual inspection. We see at once that 5493945 is a palindrome. In this study we will also refer to this type of palindromes as *regular palindromes* since we will later define another type of palindromes.

As we have seen, palindromes are easily identified by visual inspection, something we will have difficulties to do with, say prime numbers. Nevertheless, we need an algorithm to identify palindromes because we can not use our visual inspection method on integers that occur in computer analysis of various sets of numbers. The following routine, written in Ubasic, is built into various computer programs in this study:

```
10 'Palindrome identifier, Henry Ibstedt, 031021
20 input " N";N
30 s=n\10 :r=res
40 while s>0
50 s=s\10 :r=10*r+res
60 wend
70 print n,r
80 end
```

This technique of reversing a number is quite different from what will be needed later on in this study. Although very simple and useful it is worth thinking about other methods depending on the nature of the set of numbers to be examined. Let's look at prime number palindromes.

2. Prime Number Palindromes

We can immediately list the prime number palindromes which are less than 100, they are: 2, 3, 5, 7 and 11. We realize that the last digit of any prime number except 2 must be 1, 3, 7 or 9. A three digit prime number palindrome must therefore be of the types: 1x1, 3x3, 7x7 or 9x9 where xε{0, 1, …, 9}. Here, numbers have been expressed in concatenated form. When there is no risk of misunderstanding we will simply write 2x2, otherwise concatenation will be expressed 2_x_2 while multiplication will be made explicit by $2\cdot x\cdot 2$.

In explicit form we write the above types of palindromes: 101+10x, 303+10x, 707+10x and 909+10x respectively.

A 5-digit palindrome axyxa can be expressed in the form:

a_000_a+x·1010+y·100 where aε{1, 3, 7, 9}, xε{0, 1, …, 9} and yε{0, 1, …, 9}

This looks like complicating things. But not so. Implementing this in a Ubasic program will enable us to look for which palindromes are primes instead of looking for which primes are palindromes. Here is the corresponding computer code (C5):

```
 10    'Classical 5-digit Prime Palindromes (C5)
 20    'October 2003, Henry Ibstedt
 30    dim V(4),U(4)
 40    for I=1 to 4 :read V(I):next
 50    data 1,3,7,9
 60    T=10001
 70    for I=1 to 4
 80    U=0:'Counting prime palindromes
 90    A=V(I)*T
100    for J=0 to 9
110    B=A+1010*J
120    for K=0 to 9
130    C=B+100*K
140    if nxtprm(C-1)=C then print C :inc U
150    next :next
160    U(I)=U
```

```
170   next
180   for I=1 to 4 :print U(I):next
190   end
```

Before implementing this code the following theorem will be useful.

Theorem: A palindrome with an even number of digits is divisible by 11.

Proof: We consider a palindrome with 2n digits which we denote $x_1, x_2, \ldots x_n$. Using concatenation we write the palindrome

$$A=x_1x_2\ldots x_nx_n\ldots x_2x_1$$

We express A in terms of $x_1, x_2, \ldots x_n$ in the following way:

$$A=x_1(10^{2n-1}+1)+x_2(10^{2n-2}+10)+x_3(10^{2n-3}+10^2)+\ldots x_n(10^{2n-n}+10^{n-1})$$

or

$$A=\sum_{k=1}^{n} x_k(10^{2n-k}+10^{k-1}) \qquad (1)$$

We will now use the following observation:

$$10^q-1\equiv 0 \pmod{11} \text{ for } q\equiv 0 \pmod 2$$

and

$$10^q+1\equiv 0 \pmod{11} \text{ for } q\equiv 1 \pmod 2$$

We re-write (1) in the form:

$A=\sum_{k=1}^{n} x_k(10^{2n-k}\pm 1+10^{k-1}\mp 1)$ where the upper sign applies if $k\equiv 1 \pmod 2$ and the lower sign if $k\equiv 0 \pmod 2$.

From this we see that $A\equiv 0 \pmod{11}$ for $n\equiv 0 \pmod 2$.

Corollary: From this theorem we learn that the only prime number palindrome with an even number of digits is 11.

This means that we only need to examine palindromes with an odd number of digits for primality. Changing a few lines in the computer code C5 we obtain computer codes (C3, C7 and C9) which will allow us to identify all prime number palindromes less than 10^{10} in less than 5 minutes. The number of prime number palindromes in each interval was registered in a file. The result is displayed in table 1.

Table 1. Number of prime number palindromes

Number of digits		Number of palindromes of type			
	1......1	3......3	7......7	9......9	Total
3	5	4	4	2	15
5	26	24	24	19	93
7	190	172	155	151	668

9	1424	1280	1243	1225	5172

Table 2. Three-digit prime number palindromes
(Total 15)

Interval	Prime Number Palindromes				
100-199	101	131	151	181	191
300-399	313	353	373	383	
700-799	727	757	787	797	
900-999	919	929			

Table 3. Five-digit prime number palindromes
(Total 93)

10301	10501	10601	11311	11411	12421	12721	12821	13331
13831	13931	14341	14741	15451	15551	16061	16361	16561
16661	17471	17971	18181	18481	19391	19891	19991	
30103	30203	30403	30703	30803	31013	31513	32323	32423
33533	34543	34843	35053	35153	35353	35753	36263	36563
37273	37573	38083	38183	38783	39293			
70207	70507	70607	71317	71917	72227	72727	73037	73237
73637	74047	74747	75557	76367	76667	77377	77477	77977
78487	78787	78887	79397	79697	79997			
90709	91019	93139	93239	93739	94049	94349	94649	94849
94949	95959	96269	96469	96769	97379	97579	97879	98389
98689								

Table 4. Seven-digit prime number palindromes
(Total 668)

1003001	1008001	1022201	1028201	1035301	1043401	1055501	1062601
1065601	1074701	1082801	1085801	1092901	1093901	1114111	1117111
1120211	1123211	1126211	1129211	1134311	1145411	1150511	1153511
1160611	1163611	1175711	1177711	1178711	1180811	1183811	1186811
1190911	1193911	1196911	1201021	1208021	1212121	1215121	1218121
1221221	1235321	1242421	1243421	1245421	1250521	1253521	1257521
1262621	1268621	1273721	1276721	1278721	1280821	1281821	1286821
1287821	1300031	1303031	1311131	1317131	1327231	1328231	1333331
1335331	1338331	1343431	1360631	1362631	1363631	1371731	1374731
1390931	1407041	1409041	1411141	1412141	1422241	1437341	1444441
1447441	1452541	1456541	1461641	1463641	1464641	1469641	1486841
1489841	1490941	1496941	1508051	1513151	1520251	1532351	1535351
1542451	1548451	1550551	1551551	1556551	1557551	1565651	1572751
1579751	1580851	1583851	1589851	1594951	1597951	1598951	1600061
1609061	1611161	1616161	1628261	1630361	1633361	1640461	1643461
1646461	1654561	1657561	1658561	1660661	1670761	1684861	1685861
1688861	1695961	1703071	1707071	1712171	1714171	1730371	1734371
1737371	1748471	1755571	1761671	1764671	1777771	1793971	1802081
1805081	1820281	1823281	1824281	1826281	1829281	1831381	1832381
1842481	1851581	1853581	1856581	1865681	1876781	1878781	1879781
1880881	1881881	1883881	1884881	1895981	1903091	1908091	1909091
1917191	1924291	1930391	1936391	1941491	1951591	1952591	1957591
1958591	1963691	1968691	1969691	1970791	1976791	1981891	1982891
1984891	1987891	1988891	1993991	1995991	1998991		
3001003	3002003	3007003	3016103	3026203	3064603	3065603	3072703

3073703	3075703	3083803	3089803	3091903	3095903	3103013	3106013
3127213	3135313	3140413	3155513	3158513	3160613	3166613	3181813
3187813	3193913	3196913	3198913	3211123	3212123	3218123	3222223
3223223	3228223	3233323	3236323	3241423	3245423	3252523	3256523
3258523	3260623	3267623	3272723	3283823	3285823	3286823	3288823
3291923	3293923	3304033	3305033	3307033	3310133	3315133	3319133
3321233	3329233	3331333	3337333	3343433	3353533	3362633	3364633
3365633	3368633	3380833	3391933	3392933	3400043	3411143	3417143
3424243	3425243	3427243	3439343	3441443	3443443	3444443	3447443
3449443	3452543	3460643	3466643	3470743	3479743	3485843	3487843
3503053	3515153	3517153	3528253	3541453	3553553	3558553	3563653
3569653	3586853	3589853	3590953	3591953	3594953	3601063	3607063
3618163	3621263	3627263	3635363	3643463	3646463	3670763	3673763
3680863	3689863	3698963	3708073	3709073	3716173	3717173	3721273
3722273	3728273	3732373	3743473	3746473	3762673	3763673	3765673
3768673	3769673	3773773	3774773	3781873	3784873	3792973	3793973
3799973	3804083	3806083	3812183	3814183	3826283	3829283	3836383
3842483	3853583	3858583	3863683	3864683	3867683	3869683	3871783
3878783	3893983	3899983	3913193	3916193	3918193	3924293	3927293
3931393	3938393	3942493	3946493	3948493	3964693	3970793	3983893
3991993	3994993	3997993	3998993				
7014107	7035307	7036307	7041407	7046407	7057507	7065607	7069607
7073707	7079707	7082807	7084807	7087807	7093907	7096907	7100017
7114117	7115117	7118117	7129217	7134317	7136317	7141417	7145417
7155517	7156517	7158517	7159517	7177717	7190917	7194917	7215127
7226227	7246427	7249427	7250527	7256527	7257527	7261627	7267627
7276727	7278727	7291927	7300037	7302037	7310137	7314137	7324237
7327237	7347437	7352537	7354537	7362637	7365637	7381837	7388837
7392937	7401047	7403047	7409047	7415147	7434347	7436347	7439347
7452547	7461647	7466647	7472747	7475747	7485847	7486847	7489847
7493947	7507057	7508057	7518157	7519157	7521257	7527257	7540457
7562657	7564657	7576757	7586857	7592957	7594957	7600067	7611167
7619167	7622267	7630367	7632367	7644467	7654567	7662667	7665667
7666667	7668667	7669667	7674767	7681867	7690967	7693967	7696967
7715177	7718177	7722277	7729277	7733377	7742477	7747477	7750577
7758577	7764677	7772777	7774777	7778777	7782877	7783877	7791977
7794977	7807087	7819187	7820287	7821287	7831387	7832387	7838387
7843487	7850587	7856587	7865687	7867687	7868687	7873787	7884887
7891987	7897987	7913197	7916197	7930397	7933397	7935397	7938397
7941497	7943497	7949497	7957597	7958597	7960697	7977797	7984897
7985897	7987897	7996997					
9002009	9015109	9024209	9037309	9042409	9043409	9045409	9046409
9049409	9067609	9073709	9076709	9078709	9091909	9095909	9103019
9109019	9110119	9127219	9128219	9136319	9149419	9169619	9173719
9174719	9179719	9185819	9196919	9199919	9200029	9209029	9212129
9217129	9222229	9223229	9230329	9231329	9255529	9269629	9271729
9277729	9280829	9286829	9289829	9318139	9320239	9324239	9329239
9332339	9338339	9351539	9357539	9375739	9384839	9397939	9400049
9414149	9419149	9433349	9439349	9440449	9446449	9451549	9470749
9477749	9492949	9493949	9495949	9504059	9514159	9526259	9529259
9547459	9556559	9558559	9561659	9577759	9583859	9585859	9586859
9601069	9602069	9604069	9610169	9620269	9624269	9626269	9632369
9634369	9645469	9650569	9657569	9670769	9686869	9700079	9709079
9711179	9714179	9724279	9727279	9732379	9733379	9743479	9749479

9752579	9754579	9758579	9762679	9770779	9776779	9779779	9781879
9782879	9787879	9788879	9795979	9801089	9807089	9809089	9817189
9818189	9820289	9822289	9836389	9837389	9845489	9852589	9871789
9888889	9889889	9896989	9902099	9907099	9908099	9916199	9918199
9919199	9921299	9923299	9926299	9927299	9931399	9932399	9935399
9938399	9957599	9965699	9978799	9980899	9981899	9989899	

Of the 5172 nine-digit prime number palindromes only a few in the beginning and at the end of each type are shown in table 5.

Table 5a. Nine-digit prime palindromes of type 1__1
(Total 1424)

100030001	100050001	100060001	100111001	100131001	100161001
100404001	100656001	100707001	100767001	100888001	100999001
101030101	101060101	101141101	101171101	101282101	101292101
101343101	101373101	101414101	101424101	101474101	101595101
101616101	101717101	101777101	101838101	101898101	101919101
101949101	101999101	102040201	102070201	102202201	102232201
102272201	102343201	102383201	102454201	102484201	102515201
102676201	102686201	102707201	102808201	102838201	103000301
103060301	103161301	103212301	103282301	103303301	103323301
103333301	103363301	103464301	103515301	103575301	103696301
195878591	195949591	195979591	196000691	196090691	196323691
196333691	196363691	196696691	196797691	196828691	196878691
197030791	197060791	197070791	197090791	197111791	197121791
197202791	197292791	197343791	197454791	197525791	197606791
197616791	197868791	197898791	197919791	198040891	198070891
198080891	198131891	198292891	198343891	198353891	198383891
198454891	198565891	198656891	198707891	198787891	198878891
198919891	199030991	199080991	199141991	199171991	199212991
199242991	199323991	199353991	199363991	199393991	199494991
199515991	199545991	199656991	199767991	199909991	199999991

Table 5b. Nine-digit prime palindromes of type 3__3
(Total 1280)

300020003	300080003	300101003	300151003	300181003	300262003
300313003	300565003	300656003	300808003	300818003	300848003
300868003	300929003	300959003	301050103	301111103	301282103
301434103	301494103	301555103	301626103	301686103	301818103
301969103	302030203	302070203	302202203	302303203	302313203
302333203	302343203	302444203	302454203	302525203	302535203
302555203	302646203	302676203	302858203	302898203	302909203
303050303	303121303	303161303	303272303	303292303	303373303
303565303	303616303	303646303	303757303	303878303	303929303
303979303	304050403	304090403	304131403	304171403	304191403
394191493	394212493	394333493	394494493	394636493	394696493
394767493	395202593	395303593	395363593	395565593	395616593
395717593	395727593	395868593	395898593	396070693	396191693
396202693	396343693	396454693	396505693	396757693	396808693
396919693	396929693	397141793	397242793	397333793	397555793
397666793	397909793	398040893	398111893	398151893	398232893

398252893	398363893	398414893	398474893	398616893	398666893
398676893	398757893	398838893	398898893	399070993	399191993
399262993	399323993	399464993	399484993	399575993	399595993
399616993	399686993	399707993	399737993	399767993	399878993

Table 5c. Nine-digit prime palindromes of type 7__7
(Total 1243)

700020007	700060007	700090007	700353007	700363007	700404007
700444007	700585007	700656007	700666007	700717007	700737007
700848007	700858007	700878007	700989007	701000107	701141107
701151107	701222107	701282107	701343107	701373107	701393107
701424107	701525107	701595107	701606107	701636107	701727107
701747107	701838107	701919107	701979107	701999107	702010207
702070207	702080207	702242207	702343207	702434207	702515207
702575207	702626207	702646207	702676207	702737207	702767207
702838207	702919207	702929207	702989207	703000307	703060307
703111307	703171307	703222307	703252307	703393307	703444307
795848597	795878597	796060697	796080697	796222697	796252697
796353697	796363697	796474697	796494697	796515697	796636697
796666697	796707697	796717697	796747697	796848697	796939697
797262797	797363797	797393797	797444797	797525797	797595797
797676797	797828797	797898797	797939797	797949797	798040897
798181897	798191897	798212897	798292897	798373897	798383897
798454897	798535897	798545897	798646897	798676897	798737897
798797897	798818897	798838897	798919897	798989897	799050997
799111997	799131997	799323997	799363997	799383997	799555997
799636997	799686997	799878997	799888997	799939997	799959997

Table5 d. Nine-digit prime palindromes of type 9__9
(Total 1225)

900010009	900050009	900383009	900434009	900484009	900505009
900515009	900565009	900757009	900808009	900838009	900878009
900919009	900929009	901060109	901131109	901242109	901252109
901272109	901353109	901494109	901585109	901606109	901626109
901656109	901686109	901696109	901797109	901929109	901969109
902151209	902181209	902232209	902444209	902525209	902585209
902757209	902828209	902888209	903020309	903131309	903181309
903292309	903373309	903383309	903424309	903565309	903616309
903646309	903727309	903767309	903787309	903797309	903878309
903979309	904080409	904090409	904101409	904393409	904414409
994969499	995070599	995090599	995111599	995181599	995303599
995343599	995414599	995555599	995696599	995757599	995777599
996020699	996101699	996121699	996181699	996242699	996464699
996494699	996565699	996626699	996656699	996686699	996808699
996818699	996878699	996929699	996949699	996989699	997030799
997111799	997393799	997464799	997474799	997555799	997737799
997818799	997909799	997969799	998111899	998121899	998171899
998202899	998282899	998333899	998565899	998666899	998757899
998898899	998939899	998979899	999070999	999212999	999272999
999434999	999454999	999565999	999676999	999686999	999727999

An idea about the strange distribution of prime number palindromes is given in diagram 1. In fact the prime number palindromes are spread even thinner than the diagram makes believe because the horizontal scale is in interval numbers not in decimal numbers, i.e. (100-200) is given the same length as $(1.1\cdot 10^9 - 1.2\cdot 10^9)$.

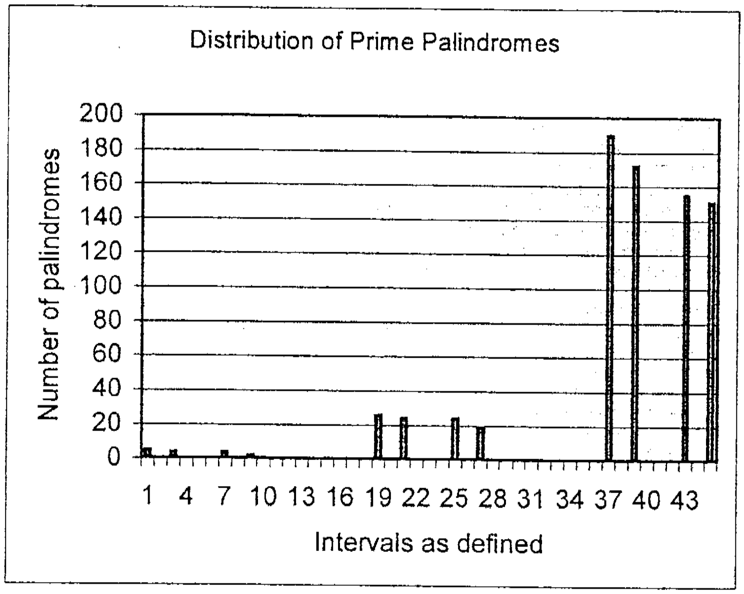

Diagram 1

Intervals 1-9: 3-digit numbers divided into 9 equal intervals.
Intervals 11-18: 4-digit numbers divided into 9 equal intervals
Intervals 19-27: 5-digit numbers divided into 9 equal intervals
Intervals 28-36: 6-digit numbers divided into 9 equal intervals
Intervals 37-45: 7-digit numbers divided into 9 equal intervals

3. Smarandache Generalized Palindromes

Definition: A Smarandache Generalized Palindrome (SGP) is any integer of the form

$x_1x_2x_3 \ldots x_n \ldots x_3x_2x_1$ or $x_1x_2x_3 \ldots x_nx_n \ldots x_3x_2x_1$

where x_1, x_2, x_3,...x_n are natural numbers. In the first case we require $n>1$ since otherwise every number would be a SGP.

Briefly speaking $x_k \varepsilon \{0,1,2, \ldots 9\}$ has been replaced by $x_k \varepsilon$ N (where N is the set of natural numbers).

Addition: To avoid that the same number is described as a SGP in more than one way this study will require the x_k to be maximum as a first priority and n to be maximum as a second priority (cf. examples below).

Interpretations and examples: Any regular palindrome (RP) is a Smarandache Generalized Palindrome (SGP), i.e. $\{RP\} \subset \{SGP\}$.
3 is a RP and also a SGP
123789 is neither RP nor SGP
123321 is RP as well as SGP

123231 is not a RP but it is a SGP 1_23_23_1
The SGP 334733 can be written in three ways: 3_3_47_3_3, 3_3473_3 and 33_47_33. Preference will be given to 33_47_33, (in compliance with the addition to the definition).
780978 is a SGP 78_09_78, i.e. we will permit natural numbers with leading zeros when they occur inside a GSP.

How do we identify a GSP generated by some sort of a computer application where we can not do it by visual inspection? We could design and implement an algorithm to identify GSPs directly. But it would of course be an advantage if methods applied in the early part of this study to identify the RPs could be applied first followed by a method to identify the GSPs which are not RPs. Even better we could set this up in such a way that we leave the RPs out completely. This leads to us to define in an operational way those GSPs which are not RPs, let us call them Extended Palindromes (EP). The set of EPs must fill the condition

$$\{RP\} \cup \{EP\} = \{GSP\}$$

4. Extended Palindromes

Definition: An Extended Palindrome (EP) is any integer of the form

$$x_1x_2x_3\ldots x_n\ldots x_3x_2x_1 \text{ or } x_1x_2x_3\ldots x_nx_n\ldots x_3x_2x_1$$

where $x_1, x_2, x_3, \ldots x_n$ are natural numbers of which at least one is greater than or equal to 10 or has one or more leading zeros. x_1 is not allowed to not have leading zeros. Again x_k should be maximum as a first priority and n maximum as a second priority.

Computer Identification of EPs

The number A to be examined is converted to a string S of length L (leading blanks are removed first). The symbols composing the string are compared by creating substrings from left L_1 and right R_1. If L_1 and R_1 are found so that $L_1 = R_1$ then A is confirmed to be an EP. However, the process must be continued to obtain a complete split of the string into substrings as illustrated in diagram 2.

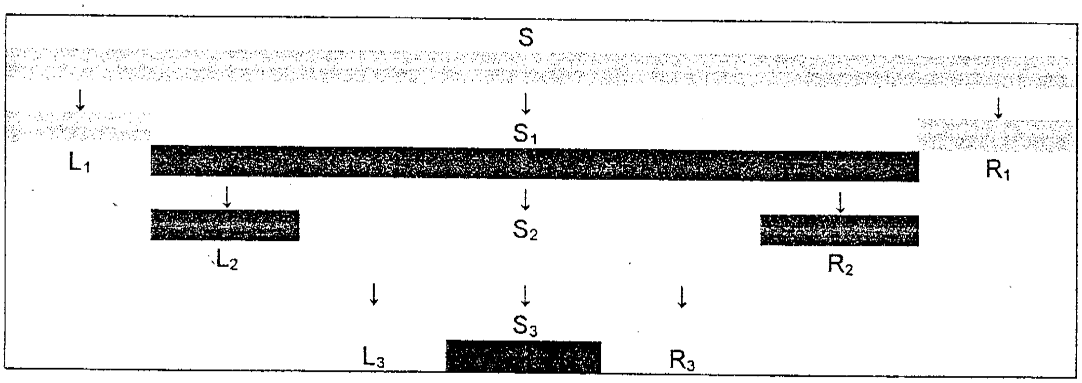

Diagram 2

Diagram 2 illustrates the identification of extended palindromes up to a maximum of 4 elements. This is sufficient for our purposes since a 4 element extended palindrome must have a minimum of 8 digits. A program for identifying extended palindromes corresponding to diagram 2 is given below. Since we have $L_k=R_k$ we will use the notation Z_k for these in the program. The program will operate on strings and the deconcatenation into extended palindrome elements will be presented as strings, otherwise there would be no distinction between 690269 and 692269 which would both be presented as 69_2 (only distinct elements will be recorded) instead of 69_02 and 69_2 respectively.

Comments on the program

It is assumed that the programming in basic is well known. Therefore only the main structure and the flow of data will be commented on:

Lines 20 – 80: Feeding the set of numbers to be examined into the program. In the actual program this is a sequence of prime numbers in the interval $a_1<a<a_2$.

Lines 90 – 270: On line 130 A is sent off to a subroutine which will exclude A if it happens to be a regular palindrome. The routine will search sub-strings from left and right. If no equal substrings are found it will return to the feeding loop otherwise it will print A and the first element Z_1 while the middle string S_1 will be sent of to the next routine (lines 280 – 400). The flow of data is controlled by the status of the variable u and the length of the middle string.

Lines 280 – 400: This is more or less a copy of the above routine. S_1 will be analyzed in the same way as S in the previous routine. If no equal substrings are found it will print S_1 otherwise it will print Z_2 and send S_2 to the next routine (lines 410 – 520).

Lines 410 – 520: This routine is similar to the previous one except that it is equipped to terminate the analysis. It is seen that routines can be added one after the other to handle extended palindromes with as many elements as we like. The output from this routine consists in writing the terminal elements, i.e. S_2 if A is a 3-element extended palindrome and Z_3 and S_3 if A is a 4-element extended palindrome.

Lines 530 – 560: Regular palindrome identifier described earlier.

```
10    'EPPRSTR, 031028
20    input "Search interval a1 to a2:";A1,A2
30    A=A1
40    while A<A2
50    A=nxtprm(A)
60    gosub 90
70    wend
80    end
90    S=str(A)
100   M=len(S)
110   if M=2 then goto 270
120   S=right(S,M-1)
130   U=0:gosub 530
140   if U=1 then goto 270
150   I1=int((M-1)/2)
160   U=0
170   for I=1 to I1
```

```
180  if left(S,I)=right(S,I) then
190  :Z1=left(S,I)
200  :M1=M-1-2*I:S1=mid(S,I+1,M1)
210  :U=1
220  endif
230  next
240  if U=0 then goto 270
250  print A;" ";Z1;
260  if M1>0 then gosub 280
270  return
280  I2=int(M1/2)
290  U=0
300  for J=1 to I2
310  if left(S1,J)=right(S1,J) then
320  :Z2=left(S1,J)
330  :M2=M1-2*J:S2=mid(S1,J+1,M2)
340  :U=1
350  endif
360  next
370  if U=0 then print " ";S1:goto 400
380  print " ";Z2;
390  if M2>0 then gosub 410 else print
400  return
410  I3=int(M2/2)
420  U=0
430  for K=1 to I3
440  if left(S2,K)=right(S2,K) then
450  :Z3=left(S2,K)
460  :M3=M2-2*K:S3=mid(S2,K+1,M3)
470  :U=1
480  endif
490  next
500  if U=0 then print " ";S2:goto 520
510  print " ";Z3;" ";S3
520  return
530  T=""
540  for I=M to 1 step -1:T=T+mid(S,I,1):next
550  if T=S then U=1:'print "a=";a;"is a RP"
560  return
```

5. Extended Prime Number Palindromes

The computer program for identification of extended palindromes has been implemented to find extended prime number palindromes. The result is shown in tables 7 to 9 for prime numbers $< 10^7$. In these tables the first column identifies the interval in the following way: $1 - 2$ in the column headed x 10 means the interval $1 \cdot 10$ to $2 \cdot 10$. EP stands for the number of extended prime number palindromes, RP is the number regular prime number palindromes and P is the number of prime numbers. As we have already concluded the first extended prime palindromes occur for 4-digit numbers and we see that primes which begin and end with one of the digits 1, 3, 7 or 9 are favored. In table 8 the pattern of behavior becomes more explicit. Primes with an even number of digits are not regular palindromes while extended prime palindromes occur for even as well as odd digit primes. It is easy to estimate from the tables that about 25% of the primes of types 1…1, 3…3, 7…7 and 9…9 are extended prime palindromes. There are 5761451 primes less than 10^8, of these 698882 are extended palindromes and only 604 are regular palindromes.

Table 7. Extended and regular palindromes
Intervals 10 -100, 100 – 1000 and 1000 -10000

x 10	EP	RP	P	x 10^2	EP	RP	P	x 10^3	EP	RP	P
1 - 2	0	1	4	1 - 2	0	5	21	1 – 2	33		135
2 - 3	0		2	2 - 3	0		16	2 – 3	0		127
3 - 4	0		2	3 - 4	0	4	16	3 – 4	28		120
4 - 5	0		3	4 - 5	0		17	4 – 5	0		119
5 - 6	0		2	5 - 6	0		14	5 – 6	0		114
6 - 7	0		2	6 - 7	0		16	6 - 7	0		117
7 - 8	0		3	7 - 8	0	4	14	7 - 8	30		107
8 - 9	0		2	8 - 9	0		15	8 - 9	0		110
9 - 10	0		1	9 - 10	0	2	14	9 - 10	27		112

Table 8. Extended and regular palindromes
Intervals 10^4 -10^5 and 10^5 – 10^6

x 10^4	EP	RP	P	X 10^5	EP	RP	P
1 - 2	242	26	1033	1 - 2	2116		8392
2 - 3	12		983	2 - 3	64		8013
3 - 4	230	24	958	3 - 4	2007		7863
4 - 5	9		930	4 - 5	70		7678
5 - 6	10		924	5 - 6	70		7560
6 - 7	9		878	6 - 7	69		7445
7 - 8	216	24	902	7 - 8	1876		7408
8 - 9	10		876	8 - 9	63		7323
9 - 10	203	19	879	9 - 10	1828		7224

Table 9. Extended and regular palindromes
Intervals 10^5 -10^6 and 10^6 – 10^7

x 10^6	EP	RP	P	x 10^7	EP	RP	P
1 - 2	17968	190	70435	1 - 2	156409		606028
2 - 3	739		67883	2 - 3	6416		587252
3 - 4	16943	172	66330	3 - 4	148660		575795
4 - 5	687		65367	4 - 5	6253		567480
5 - 6	725		64336	5 - 6	6196		560981
6 - 7	688		63799	6 - 7	6099		555949
7 - 8	16133	155	63129	7 - 8	142521		551318
8 - 9	694		62712	8 - 9	6057		547572
9 - 10	15855	151	62090	9 - 10	140617		544501

We recall that the sets of regular palindromes and extended palindromes together form the set of Smarandache Generalized Palindromes. Diagram 3 illustrates this for 5-digit primes.

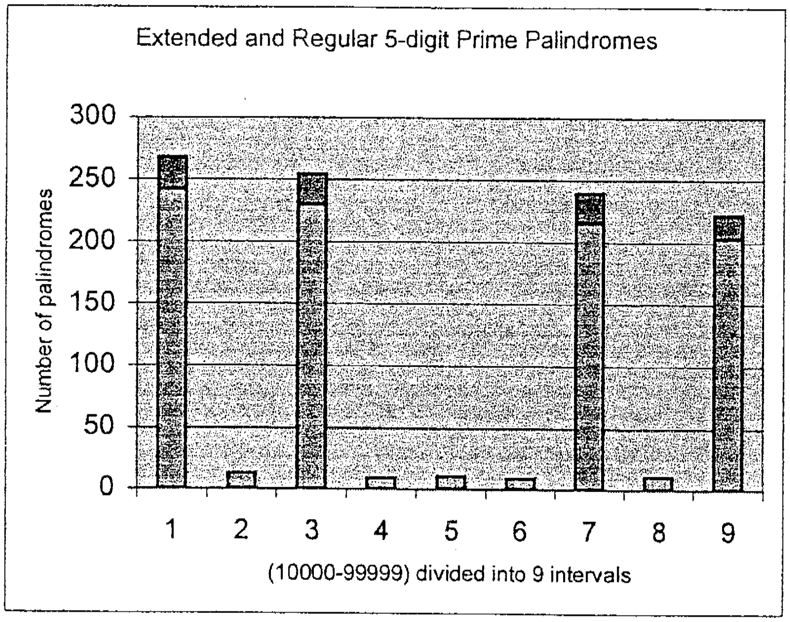

Diagram 3. Extended palindromes shown with blue color, regular with red.

Part II of this study is planned to deal with palindrome analysis of other number sequences.

References:

[1] F. Smarandache, Generalized Palindromes, Arizona State University Special Collections, Tempe.