PRIMES IN THE SMARANDACHE SQUARE PRODUCT SEQUENCE

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Abstract. For any positive integer n, let a_n be the n-th square number, and let $s_n = 1+a_1 a_2 \ldots a_n$. In this paper we prove that if $n>2$, $2|n$ and $2n+1$ is a prime, then s_n is not a prime.

For any positive integer n, let a_n be the n-th square number, and let $s_n = 1+a_1 a_2 \ldots a_n$. Then the sequence $S=\{s_n\}_{n=1}^{\infty}$ is called the Smarandache square product sequence. In[2], Iacobescu asked the following question.

Question. How many terms in S are primes?

In this paper we prove the following result:

Theorem. If $n>2$, $2|n$ and $2n+1$ is a prime, then s_n is not a prime.

Proof. By the definition of s_n, we have

(1) $s_n = 1+a_1 a_2 \ldots a_n = 1+(n!)^2$.

Let $p=2n+1$. It is a well known fact that if $2|n$ and p is p is a prime, then we have

(2) $(n!)^2 \equiv -1 \pmod p$,

(see[1,p.88]). Therefore, by (1) and (2), we get

(3) $p|s_n$.

Further, if $n>2$, then $s_n = 1+(n!)^2 > 2n+1 = p$. Thus, by (3), s_n is not a prime. The theorem is proved.

Reference

1. G.H.Hardy and e.m.Wright, An Introduction to the Theory of numbers, Oxford Univ. Press, Oxford, 1938.
2. F.Iacobescu, Smarandache pertition type and other sequences, Bull. Pure Appl. sci. Sect. E 16(1997), No.2, 237-240.