PROOF OF THE DEPASCALISATION THEOREM

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In [1] we have defined Pascalisation as follows:
Let $b_1, b_2, \ldots$ be a base sequence. Then the **Smarandache Pascal derived sequence**
$d_1, d_2, \ldots$ is defined as
$d_1 = b_1$
$d_2 = b_1 + b_2$
$d_3 = b_1 + 2b_2 + b_3$
$d_4 = b_1 + 3b_2 + 3b_3 + b_4$
...

$$d_{n+1} = \sum_{k=0}^{n} {}^{n}C_k \cdot b_{k+1}$$

Now Given S_d the task ahead is to find out the base sequence S_b. We call the process of extracting the base sequence from the Pascal derived sequence as **Depascalsation.** The interesting observation is that this again involves the Pascal's triangle, but with a difference.

On expressing b_k's in terms of d_k's We get
$b_1 = d_1$
$b_2 = -d_1 + d_2$
$b_3 = d_1 - 2d_2 + d_3$
$b_4 = -d_1 + 3d_2 - 3d_3 + d_4$
...
which suggests the possibility of

$$b_{n+1} = \sum_{k=0}^{n} (-1)^{n+k} \cdot {}^{n}C_k \cdot d_{k+1}$$

This I call as Depascalisation Theorem.
PROOF: We shall prove it by induction.
Let the proposition be true for all the numbers $1 \le k+1$. Then we have
$b_{k+1} = {}^kC_0 (-1)^{k+2} d_1 + {}^kC_1 (-1)^{k+1} d_2 + \ldots + {}^kC_k (-1)^2$
Also we have
$d_{k+2} = {}^{k+1}C_0\, b_1 + {}^{k+1}C_1\, b_2 + \ldots + {}^{k+1}C_r\, b_{r+1} + \ldots + {}^{k+1}C_{k+1}\, b_{k+2}$, which gives
$b_{k+2} = (-1)\, {}^{k+1}C_0\, b_1 - {}^{k+1}C_1\, b_2 - \ldots - {}^{k+1}C_r\, b_{r+1} - \ldots + d_{k+2}$
substituting the values of $b_1, b_2, \ldots$etc. in terms of $d_1, d_2, \ldots$, we get the coefficient of d_1 as
$(-1)\, {}^{k+1}C_0 + (-{}^{k+1}C_1)(-{}^1C_0) + (-{}^{k+1}C_2)({}^2C_0) + \ldots + (-1)^r \cdot {}^{k+1}C_r)({}^rC_0) + \ldots + (-1)^{k+1}({}^{k+1}C_k)({}^kC_0)$
$- {}^{k+1}C_0 + {}^{k+1}C_1 \cdot {}^1C_0 - {}^{k+1}C_2 \cdot {}^2C_0 + \ldots + (-1)^r \cdot {}^{k+1}C_r \cdot {}^rC_0 + \ldots + (-1)^{k+1} \cdot {}^{k+1}C_k \cdot {}^kC_0$
similarly the coefficient of d_2 is
${}^{k+1}C_1 \cdot {}^1C_1 + {}^{k+1}C_2 \cdot {}^2C_1 + \ldots + (-1)^{r+1} \cdot {}^{k+1}C_r \cdot {}^rC_1 + \ldots + (-1)^{k+1} \cdot {}^{k+1}C_k \cdot {}^kC_1$
on similar lines we get the coefficient of d_{m+1} as

$^{k+1}C_m . {}^mC_m + {}^{k+1}C_{m+1} . {}^{m+1}C_m - \ldots + (-1)^{r+m} . {}^{k+1}C_{r+m} . {}^{r+m}C_m + \ldots + (-1)^{k+m} . {}^{k+1}C_k . {}^kC_m$

$$= \sum_{h=0}^{k-m} (-1)^{h+1}\, {}^{k+1}C_{m+h} . {}^{m+h}C_m$$

$$\sum_{h=0}^{(k+1)-m} (-1)^{h+1}\, {}^{k+1}C_{m+h} . {}^{m+h}C_m \quad \div \quad (-1)^{k+m} . {}^{k+1}C_{k+1} . {}^{k+1}C_m \qquad (1)$$

Applying theorem {4.2} of reference [2], in (1) we get

$= {}^{k+1}C_m \{ 1 + (-1)\}^{k+1-m} + (-1)^{k+m} . {}^{k+1}C_m$

$= (-1)^{k+m} . {}^{k+1}C_m$

which shows that the proposition is true for (k+2) as well. The proposition has already been verified for k+1 = 3 , hence by induction the proof is complete.

In matrix notation if we write

$[b_1, b_2 , \ldots b_n]_{1xn} * [p_{i,j}]'_{nxn} = [d_1, d_2 , \ldots d_n]_{1xn}$

where $[p_{i,j}]'_{nxn}$ = the transpose of $[p_{i,j}]_{nxn}$ and

$[p_{i,j}]_{nxn}$ is given by $p_{i,j} = {}^{i-1}C_{j-1}$ if $i \le j$ else $p_{i,j} = 0$

Then we get the following result

If $[q_{i,j}]_{nxn}$ is the transpose of the inverse of $[p_{i,j}]_{nxn}$ Then

$q_{i,j} = (-1)^{j+i} . {}^{i-1}C_{j-1}$

We also have

$[b_1, b_2 , \ldots b_n] * [q_{i,j}]'_{nxn} = [d_1, d_2 , \ldots d_n]$

where $[q_{i,j}]'_{nxn}$ = The Transpose of $[q_{i,j}]_{nxn}$

References:

[1] Amarnath Murthy, ' Smarandache Pascal Derived Sequences', SNJ, March ,2000.

[2] Amarnath Murthy, 'More Results and Applications of the Smarandache Star Function., SNJ, VOL.11, No. 1-2-3, 2000.