PROPOSED PROBLEM (3)

Let $\eta(n)$ be Smarandache Function: the smallest integer m such that m! is divisible by n. Calculate $\eta(p^{p+1})$, where p is an odd prime number.

Solution.

The answer is p^2, because:

$p^2! = 1 \cdot 2 \cdot \ldots \cdot p \cdot \ldots \cdot (2p) \cdot \ldots \cdot ((p-1)p) \cdot \ldots \cdot (pp)$, which is divisible by p^{p+1}.

Any another number less than p^2 will have the property that its factorial is divisible by p^k, with $k < p + 1$, but not divisible by p^{p+1}.

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