REMARK ON THE 62-th SMARANDACHE'S PROBLEM

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In [1] Florian Smarandache formulated 105 unsolved problems.

The 62-th problem is the following:

Let $1 \leq a_1 < a_2 < \ldots$ be an infinite sequence of integers such that any three members do not constitute an arithmetic progression. Is it true that always

$$\sum_{n\geq 1} \frac{1}{a_n} \leq 2?$$

In [2-4] some counterexamples are given.

Easily it can be seen that the set of numbers $\{1, 2, 4, 5, 10\}$ does not contain three numbers which are members of an arithmetic progression. On the other hand

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{4} + \frac{1}{5} + \frac{1}{10} = 2\frac{1}{20} > 2.$$

Therefore, Smarandache's problem is not true in the present form, because the sum of the members of every one sequence with the above property and with first members 1, 2, 4, 5, 10 will be bigger than 2.

Some modifications of the above problem are discussed in [3,4].

We can construct the sequence which contains the minimal possible members, satisfying the Smarandache's property. The first 100 members of this sequence are:

$$1, 2, 4, 5, 10, 11, 13, 14, 28, 29, 31, 32, 37, 38, 40, 41, 82, 83, 85, 86, 91, 92, 94, 95, 109, 110, 112,$$
$$113, 118, 119, 121, 122, 244, 245, 247, 248, 253, 254, 256, 257, 271, 272, 274, 275, 280, 281, 283,$$
$$284, 325, 326, 328, 329, 334, 335, 337, 338, 352, 353, 355, 356, 361, 362, 364, 365, 730, 731, 733,$$
$$734, 739, 740, 742, 743, 757, 758, 760, 761, 766, 767, 769, 770, 811, 812, 814, 815, 820, 821, 823,$$
$$824, 838, 839, 841, 842, 847, 848, 850, 851, 973, 974, 976, 977$$

In another paper the properties of this sequence will be discussed in details. Some of them are given in [3,4].

We must note that it was checked by a computer that the sum of the first 18567 members of the sequence (the 18567-th member is 4962316) is 3.00000013901583..., i.e. for this sequence

$$\sum_{n\geq 1} \frac{1}{a_n} > 3.$$

It can be easily seen that if the first member of the sequence satisfying the Smarandache's property is not 1, or if its second member is not 2, then

$$\sum_{n\geq 1} \frac{1}{a_n} < 3.$$

On the other hand, there are an infinite number of sequences for which

$$\sum_{n\geq 1} \frac{1}{a_n} > 2,$$

because, for example, all sequences (their number is, obviously, infinite) generated by the above one without only one of its members will satisfy the last inequality.

This number will be discussed in the next paper of ours, too.

Now we shall cite the following unsolved problem from [2]:

Given a sequence of integers $a_1 \leq a_2 \leq ... \leq a_k \leq ...$ where no three form an arithmetic progression, is there any bound on the sum

$$\sum_{n\geq 1} \frac{1}{a_n}?$$

From the above remark it follows that 3 is a bound of all sequences with the above property without the first sequence shown above. Some properties of this bound also will be discussed in the next paper of ours.

REFERENCES:

[1] F. Smarandache, Only problems, not solutions!. Xiquan Publ. House, Chicago, 1993.

[2] C. Ashbacher, Pluckings from the Tree of Smarandache Sequences and Functions. American Research Press, Lupton, 1998.

[3] K. Atanassov, On the 62-nd Smarandache's problem. Notes on Number Theory and Discrete Mathematics, Vol. 5 (1999), No. 3, 100-101.

[4] K. Atanassov, On Some of the Smarandache's Problems. American Research Press, Lupton, 1999.

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