SMARANDACHE REVERSE AUTO CORRELATED SEQUENCES AND SOME FIBONACCI DERIVED SMARANDACHE SEQUENCES

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Let a_1, a_2, a_3, . . . be a base sequence. We define a **Smarandache Reverse Auto-correlated Sequence (SRACS)** b_1, b_2, b_3, . . . as follow :

$b_1 = a^2_1$, $b_2 = 2a_1a_2$, $b_3 = a^2_2 + 2a_1a_3$, etc. by the following transformation

$$b_n = \sum_{k=1}^{n} a_k . a_{n-k+1}$$

and such a transformation as **Smarandache Reverse Auto Correlation Transformation (SRACT)**

We consider a few base sequences.

(1) 1, 2, 3, 4, 5, . . .

i.e. ${}^1C_1$, ${}^2C_1$, ${}^3C_1$, ${}^4C_1$, ${}^5C_1$, . . .

The SRACS comes out to be

1, 4, 10, 20, 35, . . . which can be rewritten as

i.e. ${}^3C_3$, ${}^4C_3$, ${}^5C_3$, ${}^6C_3$, ${}^7C_3$, . . . we can call it SRACS(1)

Taking this as the base sequence we get SRACS(2) as

1, 8, 36, 120, 330, . . . which can be rewritten as

i.e. ${}^7C_7$, ${}^8C_7$, ${}^9C_7$, ${}^{10}C_7$, ${}^{11}C_7$, ...Taking this as the base sequence we get SRACS(3) as

1, 16, 136, 816, 3876, . . .

i.e. ${}^{15}C_{15}$, ${}^{16}C_{15}$, ${}^{17}C_{15}$, ${}^{18}C_{15}$, ${}^{19}C_{15}$, . . . ,

This suggests the possibility of the following :

conjecture-I

The sequence obtained by 'n' times Smarandache Reverse Auto Correlation Transformation (SRACT) of the set of natural numbers is given by the following:

SRACS(n)

$^{h-1}C_{h-1}$, $^{h}C_{h-1}$, $^{h+1}C_{h-1}$, $^{h+2}C_{h-1}$, $^{h+3}C_{h-1}$, . . . **where $h = 2^{n+1}$.**

(2) Triangular number as the base sequence:

1, 3, 6, 10, 15, . . .

i.e. $^{2}C_{2}$, $^{3}C_{2}$, $^{4}C_{2}$, $^{5}C_{2}$, $^{6}C_{2}$, . . .

The SRACS comes out to be

1, 6, 21, 56, 126, . . . which can be rewritten as

i.e. $^{5}C_{5}$, $^{6}C_{5}$, $^{7}C_{5}$, $^{8}C_{5}$, $^{9}C_{5}$, . . . we can call it SRACS(1)

Taking this as the base sequence we get SRACS(2) as

1, 12, 78, 364, 1365, . . .

i.e. $^{11}C_{11}$, $^{12}C_{11}$, $^{13}C_{11}$, $^{14}C_{11}$, $^{15}C_{11}$, . . . ,Taking this as the base sequence we get SRACS(3) as

1, 24, 300, 2600, 17550, . . .

i.e. $^{23}C_{23}$, $^{24}C_{23}$, $^{25}C_{23}$, $^{26}C_{23}$, $^{27}C_{23}$, . . . ,

This suggests the possibility of the following

conjecture-II

The sequence obtained by 'n' times Smarandache Reverse Auto Correlation transformation (SRACT) of the set of Triangular numbers is given by

SRACS(n)

$^{h-1}C_{h-1}$, $^{h}C_{h-1}$, $^{h+1}C_{h-1}$, $^{h+2}C_{h-1}$, $^{h+3}C_{h-1}$, . . . **where $h = 3.2^{n}$.**

This can be generalised to conjecture the following:

Conjecture-III :

Given the base sequence as $^{n}C_{n}$, $^{n+1}C_{n}$, $^{n+2}C_{n}$, $^{n+3}C_{n}$, $^{n+4}C_{n}$, . . .

The SRACS(n) is given by

$^{h-1}C_{h-1}$, $^{h}C_{h-1}$, $^{h+1}C_{h-1}$, $^{h+2}C_{h-1}$, $^{h+3}C_{h-1}$, . . . **where $h = (n+1).2^{n}$.**

SOME FIBONACCI DERIVED SMARANDACHE SEQUENCES

1. Smarandache Fibonacci Binary Sequence (SFBS):

In Fibonacci Rabbit problem we start with an immature pair ' I ' which matures after one season to 'M' . This mature pair after one season stays alive and breeds a new immature pair and we get the following sequence

I→ M → MI→ M IM→ M IMMI→ MIMMIMIM→ MIMMIMIMMIMMI

If we replace I by 0 and M by 1 we get the following binary sequence

0→ 1→ 10→ 101→ 10110→ 10110101→ 1011010110110

The decimal equivalent of the above sequences is

0→ 1→ 2→ 5→ 22→ 181→ 5814

we define the above sequence as the **SFBS**

We derive a **reduction formula** for the general term:

From the binary pattern we observe that

$T_n = T_{n-1}\, T_{n-2}$ {the digits of the T_{n-2} placed to the left of the digits of T_{n-1}.}

Also the number of digits in T_r is nothing but the r^{th} Fibonacci number by definition . Hence we have

$$\mathbf{T_n = T_{n-1} \,.\, 2^{F(n-2)} + T_{n-2}}$$

Problem: 1. How many of the above sequence are primes?

2. How many of them are Fibonacci numbers?

(2)Smarandache Fibonacci product Sequence:

The Fibonacci sequence is 1, 1, 2, 3, 5, 8, . . .

Take $T_1 = 2$, and $T_2 = 3$ and then $T_n = T_{n-1} \,.\, T_{n-2}$ we get the following sequence

2, 3, 6, 18, 108, 1944, 209952 ------**(A)**

In the above sequence which is just obtained by the first two terms , the whole Fibonacci sequence is inherent. This will be clear if we rewrite the above sequence as below:

$2^1, 3^1, 2^1 3^1, 2^1 3^2, 2^2 3^3, 2^3 3^5, 2^5 3^8, \ldots$

we have $\mathbf{T_n = 2^{F_{n-1}} \,.\, 3^{F_n}}$

The above idea can be extended by choosing r terms instead of two only and define

$T_n = T_{n-1}\, T_{n-2}\, T_{n-3} \ldots T_{n-r}$ for $n > r$.

Conjecture : (1) The following sequence obtained by incrementing the sequence (A) by 1

3, 4, 7, 19, 1945, 209953 . . . contains infinitely many primes .

(2) It does not contain any Fibonacci number.