SMARANDACHE ALGEBRAIC STRUCTURES

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A few notions are introduced in algebra in order to better study the congruences. Especially the Smarandache semigroups are very important for the study of congruences.

1) The SMARANDACHE SEMIGROUP is defined to be a semigroup A such that a proper subset of A is a group (with respect with the same induced operation).
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

For example, if we consider the commutative multiplicative group
SG = {18^2, 18^3, 18^4, 18^5} (mod 60)
we get the table:

x	24	12	36	48
24	36	48	24	12
12	48	24	12	36
36	24	12	36	48
48	12	36	48	24

Unitary element is 36.

Using the Smarandache's algorithm [see 2] we get that
18^2 is congruent to 18^6 (mod 60).

Now we consider the commutative multiplicative semigroup
SS = {18^1, 18^2, 18^3, 18^4, 18^5} (mod 60)
we get the table:

x	18	24	12	36	48
18	24	12	36	48	24

24	12	36	48	24	12
12	36	48	24	12	36
36	48	24	12	36	48
48	24	12	36	48	24

Because SS contains a proper subset SG, which is a group, then SS is a Smarandache Semigroup. This is generated by the element 18. The powers of 18 form a cyclic sequence: 18, 24,12,36,48, 24,12,36,48,... .

Similarly are defined:

2) The SMARANDACHE MONOID is defined to be a monoid A such that a proper subset of A is a group (with respect with the same induced operation).
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

3) The SMARANDACHE RING is defined to be a ring A such that a proper subset of A is a field (with respect with the same induced operation).
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

We consider the commutative additive group $M=\{0,18^2,18^3,18^4,18^5\}$ (mod 60) [using the module 60 residuals of the previous powers of 18],
$M=\{0,12,24,36,48\}$, unitary additive unit is 0.
(M,+,x) is a field.
While (SR,+,x)=$\{0,6,12,18,24,30,36,42,48,54\}$ (mod 60) is a ring whose proper subset $\{0,12,24,36,48\}$ (mod 60) is a field.
Therefore (SR,+,x) (mod 60) is a Smarandache Ring.
This feels very nice.

4) The SMARANDACHE SUBRING is defined to be a Smarandache Ring B which is a proper subset of s Smarandache Ring A (with respect with the same induced operation).

5) The SMARANDACHE IDEAL is defined to be an ideal A such that a proper subset of A is a field (with respect with the same induced operation).
By proper subset we understand a set included in A, different from the

empty set, from the unit element -- if any, and from A.

6) The SMARANDACHE SEMILATTICE is defined to be a lattice A such that a proper subset of A is a lattice (with respect with the same induced operation).
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

7) The SMARANDACHE FIELD is defined to be a field (A,+,x) such that a proper subset of A is a K-algebra (with respect with the same induced operations, and an external operation).
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

8) The SMARANDACHE R-MODULE is defined to be an R-MODULE (A,+,x) such that a proper subset of A is a S-algebra (with respect with the same induced operations, and another "x" operation internal on A), where R is a commutative unitary Smarandache ring and S its proper subset field.
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

9) The SMARANDACHE K-VECTORIAL SPACE is defined to be a K-vectorial space (A,+,.) such that a proper subset of A is a K-algebra (with respect with the same induced operations, and another "x" operation internal on A), where K is a commutative field.
By proper subset we understand a set included in A, different from the empty set, from the unit element -- if any, and from A.

References:

[1] Porubsky, Stefan, "A Smarandache Form of the Fermat-Euler Theorem in Number Theory", in <Smarandache Notions Journal>, Vol. 8, No. 1-2-3, 1997, pp.7-14.
[2] Smarandache, Florentin, "A generalization of Euler's Theorem", in <Analele Univ. Timisoara>, Vol. XVIII, Fasc. 1, 1980, 79-88.
[3] Smarandache, Florentin, "Special Algebraic Structures", 1973, Arizona State University Special Collections, Tempe, AZ, USA.