SMARANDACHE CONCATENATED POWER DECIMALS AND THEIR IRRATIONALITY

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Abstract

In this paper we prove that all Smarandache concatenated k-power decimals are irrational numbers.

For any positive integer k, we define the Smarandache concatenated k-power decimal α_k as follows:

$$\alpha_1 = 0.1234567891011\ldots,\quad \alpha_2 = 0.149162536496481100121\ldots$$

(1)

$$\alpha_3 = 0.18276412521634351272910001331\ldots,\quad \ldots, \text{etc.}$$

In this peper we discuss the irrationally of α_k. We prove the following result:

Theorem. For any positive integer k, α_k is an irrational number.

Proof. We nou suppose that α_k is a rational number. Then, by [1, Theorem 135], α_k is an infinite periodical decimal such that

(2) $$\alpha_k = 0.a_1\ldots\overline{a_r\, a_{r+1}\ldots a_{r+t}},$$

were r, t are fixed integers, with $r \geq 0$ and $t > 0$, $a_1, \ldots, a_r, a_{r+1}, \ldots, a_{r+t}$ are integers satisfying $0 \leq a_i \leq 9$ $(i = 1, 2, \ldots, r+t)$.
However, we see from (1) that there exist arbitrary many continuous zeros in the expansion of α_k. Therefore, we find from (2) that $a_{r+1} = \ldots = a_{r+t} = 0$. It implies that α_k is a finite decimal; a contradiction. Thus, α_k must be an irrational number. The theorem is proved.

Finally, we pose a further question as follows:

Question. Is α_k a transcedental number for any positive integer k?

By an old result of Mahler [2], the answer of our question is positive for k=1.

References:

1. G.H.Hardy and E.M.Wright, "An Introduction to the Theory of Numbers", Oxford University Press, Oxford, 1938.
2. K.Mahler, "Aritmetische Eigenschaften einer Klasse von Dezimalbruchen", Nederl. Akad. Wetesch. Proc., Ser.A, 40 (1937), 421-428.