SMARANDACHE FACTOR PARTITIONS OF A TYPICAL CANONICAL FORM.

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION , as follows:

Let $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r$ be a set of r natural numbers and $p_1, p_2, p_3, \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r)$ is defined as the number of ways in which the number

$N = p_1^{\alpha 1} \; p_2^{\alpha 2} \; p_3^{\alpha 3} \ldots p_r^{\alpha r}$ could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 = 2$, $p_2 = 3$ etc.

In the present note we derive a formula forr the case $N = p_1^{\alpha} p_2^{2}$

DISCUSSION:

Theorem(5.1):

$$F'(p_1^{\alpha} p_2^{2}) = F(\alpha,2) = \sum_{k=0}^{\alpha} P(k) + \sum_{j=0}^{r} \sum_{i=0}^{\alpha-2j} P(i)$$

where $r = [\alpha/2]$ $\alpha = 2r$ or $\alpha = 2r + 1$

PROOF: Following are the distinct mutually exclusive and exhaustive cases. Only the numbers in the bracket [ ] are to be further decomposed.

Case I: $(p_2)\ [p_1^{\alpha} p_2^{2}]$ gives $F'^{*}(p_1^{\alpha}) = \sum_{k=0}^{\alpha} P(i)$

Case II: $\{A_1\} \rightarrow \quad (p_2^{2})\ [p_1^{\alpha}] \quad \text{-------}\rightarrow P(\alpha)$

$\{A_2\} \rightarrow \quad (p_2^{2}\ p_1)\ [p_1^{\alpha-1}] \text{-------}\rightarrow P(\alpha-1)$

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$\{A_{\alpha}\} \rightarrow \quad (p_2^{2}\ p_1^{\alpha})\ [p_1^{\alpha-\alpha}] \quad \text{-------}\rightarrow P(\alpha-\alpha) = P(0)$

Hence Case II contributes $\sum_{i=0}^{\alpha} P(i)$

Case III: $\{B_1\} \rightarrow (p_1 p_2)(p_1 p_2)\ [p_1^{\alpha-2}] \quad \text{-------}\rightarrow P(\alpha-2)$

$\{B_2\} \rightarrow (p_1 p_2)\ (p_1^{2}\ p_2)\ [p_1^{\alpha-3}] \text{-------}\rightarrow P(\alpha-3)$

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$\{B_{\alpha-2}\} \rightarrow (p_1 p_2)\ \ (p_1^{\alpha-1}\ p_2)\ [p_1^{\alpha-\alpha}] \quad \text{-------}\rightarrow P(\alpha-\alpha) = P(0)$

Hence Case III contributes $\sum_{i=0}^{\alpha-2} P(i)$

Case IV: $\{C_1\} \rightarrow (p_1^{2}\ p_2)\ (p_1^{2}\ p_2)\ [p_1^{\alpha-4}] \quad \text{-------}\rightarrow P(\alpha-4)$

$\{C_2\} \rightarrow (p_1^{2}\ p_2)(p_1^{3}\ p_2)\ [p_1^{\alpha-5}] \quad \text{-------}\rightarrow P(\alpha-5)$

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$\{C_{\alpha-4}\} \rightarrow (p_1^{2}\ p_2)(p_1^{\alpha-2}\ p_2)\ [p_1^{\alpha-\alpha}] \quad \text{-------}\rightarrow P(\alpha-\alpha) = P(0)$

Hence Case IV contributes $\sum_{i=0}^{\alpha-4} P(i)$

{ **NOTE:** The factor partition $(p_1^2 p_2)(p_1 p_2)[p_1^{\alpha-3}]$ has already been covered in case III hence is omitted in case IV. The same logic is extended to remaining (following) cases also.}

Case V: $\{D_1\} \rightarrow (p_1^3 p_2)(p_1^3 p_2)[p_1^{\alpha-6}]$ ------→ $P(\alpha-4)$

$\{D_2\} \rightarrow (p_1^3 p_2)(p_1^4 p_2)[p_1^{\alpha-7}]$ ------→ $P(\alpha-5)$

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$\{D_{\alpha-4}\} \rightarrow (p_1^3 p_2)(p_1^{\alpha-3} p_2)[p_1^{\alpha-\alpha}]$ ------→ $P(\alpha-\alpha) = P(0)$

Hence Case V contributes $\sum_{i=0}^{\alpha-6} P(i)$

On similar lines case VI contributes $\sum_{i=0}^{\alpha-8} P(i)$

we get contributions upto $\sum_{i=0}^{\alpha-2r} P(i)$

where $2r < \alpha < 2r+1$ or $r = [\alpha/2]$

summing up all the cases we get

$$F'(p_1^{\alpha} p_2^{2}) = F(\alpha,2) = \sum_{k=0}^{\alpha} P(k) + \sum_{j=0}^{r} \sum_{i=0}^{\alpha-2j} P(i)$$

where $r = [\alpha/2]$ $\quad \alpha = 2r$ or $\alpha = 2r+1$

This completes the proof of theorem (5.1).

COROLLARY:(5.1)

$$F'(p_1^{\alpha}p_2^{2}) = \sum_{k=0}^{r} (k+2)\,[\,P(\alpha-2k) + P(\alpha-2k-1)] \quad \text{-----(5.1)}$$

Proof: In theorem (5.1) consider the case $\alpha = 2r$, we have

$$F'(p_1^{2r}p_2^{2}) = F(\alpha,2) = \sum_{k=0}^{2r} P(k) + \sum_{j=0}^{r} \sum_{i=0}^{\alpha-2j} P(i) \quad \text{-------(5.2)}$$

Second term on the RHS can be expanded as follows

$$\begin{array}{l} P(\alpha) + P(\alpha-1) + P(\alpha-2) + P(\alpha-3) + \ldots + P(2) + P(1) + P(0) \\ \qquad P(\alpha-2) + P(\alpha-3) + \ldots + P(2) + P(1) + P(0) \\ . \qquad\qquad P(\alpha-4) + \ldots P(2) + P(1) + P(0) \\ . \\ . \\ \qquad\qquad\qquad P(2) + P(1) + P(0) \\ \qquad\qquad\qquad\qquad P(0) \end{array}$$

summing up column wise

$$= [P(\alpha) + P(\alpha-1)] + 2\,[P(\alpha-2) + P(\alpha-3)] + 3\,[\,P(\alpha-4) + P(\alpha-5)] + \ldots$$

$$+ (r-1)\,[P(2) + P(1)] + r\,P(0).$$

$$= \sum_{k=0}^{r} (k+1)\,[\,P(\alpha-2k) + P(\alpha-2k-1)]$$

{Here $P(-1) = 0$ has been defined.}

hence

$$F'(p_1^{\alpha}p_2^{2}) = \sum_{k=0}^{r} P(k) + \sum_{k=0}^{r} (k+1)\,[\,P(\alpha-2k) + P(\alpha-2k-1)]$$

or

$$F'(p_1^{\alpha}p_2^{2}) = \sum_{k=0}^{r} (k+2)\,[\,P(\alpha-2k) + P(\alpha-2k-1)]$$

Consider the case $\alpha = 2r+1$, the second term in the expression (5.2)

can be expanded as

$$
\begin{array}{l}
P(\alpha) + P(\alpha-1) + P(\alpha-2) + P(\alpha-3) + \ldots + P(2) + P(1) + P(0) \\
\qquad P(\alpha-2) + P(\alpha-3) + \ldots + P(2) + P(1) + P(0) \\
. \qquad P(\alpha-4) + \ldots P(2) + P(1) + P(0) \\
. \\
. \qquad P(3) + P(2) + P(1) + P(0) \\
\qquad P(1) + P(0)
\end{array}
$$

summing up column wise we get

$= [P(\alpha) + P(\alpha-1)] + 2 [P(\alpha-2) + P(\alpha-3)] + 3 [P(\alpha-4) + P(\alpha-5)] + \ldots$

$+ (r-1) [P(3) + P(2)] + r[P(1) + P(0)].$

$$= \sum_{k=0}^{r} (k+1) [P(\alpha-2k) + P(\alpha-2k-1)] , \quad \alpha = 2r+1$$

on adding the first term , we get

$$F'(p_1^{\alpha} p_2^{2}) = \sum_{k=0}^{r} (k+2) [P(\alpha-2k) + P(\alpha-2k-1)]$$

{**Note** here P(-1) shall not appear.}
Hence for all values of α we have

$$\mathbf{F'(p_1^{\alpha} p_2^{2}) = \sum_{k=0}^{[\alpha/2]} (k+2) [P(\alpha-2k) + P(\alpha-2k-1)]}$$

This completes the proof of the Corollary (5.1).

REFERENCES:

[1] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

[2] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texax at Austin, USA.

[3] 'Smarandache Notion Journal' Vol. 10 ,No. 1-2-3, Spring 1999. Number Theory Association of the UNIVERSITY OF CRAIOVA.