SMARANDACHE ITERATIONS OF THE FIRST KIND ON FUNCTIONS INVOLVING DIVISORS AND PRIME FACTORS

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ABSTRACT

In this paper we consider Smarandache Iterations of the First Kind [2] upon four new functions which deal with divisors and prime factors of positive integers, make conjectures and give some open problems.

I INTRODUCTION

Consider Smarandache Iterations of the First Kind where a function, f(n) <= n for all n, is iterated until it reaches a constant value. For example, let d(n) be the number of positive divisors of n and 2 the constant value to be reached. For n=8 we would have:

d(d(d(8))) = d(d(4)) = d(3) = 2.

Thus SI1_d(8) = 3, because it takes 3 iterations to reach the constant value 2.

Or, another way to represent this is:

8 -> 4 -> 3 -> 2; and say 8 takes three "steps" to reach 2 when iterating the function d(n).

In this paper we will drop the SI1 notation, use the "step" terminology, and also investigate some functions where f(n) is not <= n for all n.

II INVESTIGATIONS AND OPEN PROBLEMS

(A) Let f(n) be a function giving the absolute value of the largest prime factor subtracted from the largest proper divisor of a positive integer n:

f(n)=abs(lpd(n)-Lpf(n)).

(Here we take the absolute value to avoid getting negative values.) For example, when n=13, the largest proper divisor is 1 and the largest prime factor is 13, which would be:

1 - 13 = -12 and |-12| = 12, so f(13) = 12.

If we iterate the function f(n), how many iterations will it take for a given integer n to reach 0? E.g. iterating f(13) gives:

13 -> 12 -> 3 -> 2 -> 1 -> 0; 5 steps to reach 0.

Iterating f(412) gives:

412 -> 103 -> 102 -> 34 -> 0; 4 steps to reach 0.

Here is the sequence of the number of steps it takes f(n) to reach zero upon iteration for n=1 to 100 (A075660) [1]:

1,2,3,1,2,1,2,3,1,1,2,4,5,1,1,2,3,2,3,3,1,1,2,2,1,
1,2,3,4,2,3,2,1,1,1,2,3,1,1,2,3,2,3,3,2,1,2,2,1,4,
1,6,7,3,1,2,1,1,2,2,3,1,2,3,1,2,3,4,1,4,5,2,3,1,4,
4,1,2,3,2,3,1,2,2,1,1,1,2,3,3,1,3,1,1,1,3,4,3,2,3,

Now a natural question to ask is: what is the smallest number requiring k steps to reach 0 when iterating f(n)? Using the programming package PARI/GP [3] a program was written to construct the following table of these numbers for k <= 19 (A074347) [1]:

Number of steps	Smallest number
1	1
2	2
3	3
4	12
5	13
6	52
7	53
8	131
9	271
10	811
11	1601
12	2711
13	8111
14	13997
15	34589
16	74551
17	147773
18	310567
19	621227

Note that 1, 12, and 52 are the only non-prime values in the "smallest number" sequence above. Of course it is easy to see why there is an abundance of primes here. f(p)=p-1 for any prime p since the largest prime factor of p is p and the largest proper divisor of p is always 1. Because f(p) will always equal p-1 it will take more steps for f(p) to reduce to zero upon iteration.

Open problems: What is the next non-prime number in this sequence, if one exists? What is the 20th term of this sequence?

We conjecture that the above sequence: smallest number requiring k steps to reach zero when iterating the function f(n)=abs(lpd(n)-Lpf(n)), is finite. Or stated another way, there is a number K such that no number requires greater than K steps when iterating f(n) to reach zero.

(B) Next we will perform the same operation but instead of using

the largest proper divisor, we will define a function g(n) with Lpf(n) and the largest common difference between consecutive divisors when they are ordered by size, or: g(n)=Lcdd(n)-Lpf(n).
E.g. for n=9, the divisors of 9 are [1, 3, 9] with the largest difference between consecutive divisors being 9-3=6. And the largest prime factor being 3 and 6-3=3, so g(9)=3. When iterating this function it becomes apparent that every number will eventually reach 0 or -1, so we can ask for a sequence of the number of steps it takes any n to reach 0 or -1 when iterating g(n). Here are the first one hundred values (A075661) [1]:

1,1,1,1,1,1,1,2,2,1,1,2,1,1,2,2,1,2,1,2,2,1,1,3,3,
1,3,2,1,2,1,2,2,1,3,3,1,1,2,3,1,2,1,2,4,1,1,3,4,3,
2,2,1,4,3,3,2,1,1,4,1,1,4,3,3,2,1,2,2,3,1,3,1,1,5,
2,4,2,1,4,3,1,1,4,3,1,2,3,1,4,4,2,2,1,3,5,1,3,4,5,

Again, we ask the question: what is the smallest number requiring k steps for the iterated function g(n) to reach 0 or -1? Below is a table of these numbers for k <= 26 (A074348) [1]:

Number of steps	Smallest number
1	1
2	8
3	24
4	45
5	75
6	160
7	273
8	429
9	741
10	1001
11	1183
12	1547
13	2645
14	3553
15	4301
16	5423
17	10880
18	23465
19	33371
20	39109
21	49075
22	74011
23	98933
24	104371
25	107911
26	163489

Obviously none of the terms in the above sequence will be prime since the largest common difference between any prime p is p-1 and the largest prime factor of a prime is p, and (p-1)-p = -1, therefore all primes will take one step only to reach -1.
Also, the author noticed no pattern when looking at the factorizations of the "smallest number" sequence above.

Open problem: What is the value for k=27? We conjecture that this sequence is finite (although not necessarily at k=26).

(C) Now we will add a new concept to our iterative work, the concept of reversing the elements of n in the functions we have been exploring. This may seem unnatural, but let's get a little adventurous, shall we?

Consider the function we worked with in section A, except now we will reverse lpd(n) and Lpf(n):

h(n)=abs(reverse(lpd(n))-reverse(Lpf(n)))

that is, we are taking the absolute value of the reversal of the largest proper divisor of n minus the reversal of the largest prime factor of n. This is an erratic function, and notice that since h(n) is not <= n for all n:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
h(n)	0	1	2	0	4	0	6	2	0	0	10	3	30	0	0	6	70	6	90	4

It deviates slightly from the definition of Smarandache Iterations of the First Kind.

Here is the sequence of the number of steps it takes h(n) to reach zero for n=1 to 100:

1,2,3,1,2,1,2,3,1,1,2,4,3,1,1,2,3,2,3,2,1,1,6,3,1,
1,2,2,2,2,5,3,1,1,1,3,5,1,1,4,4,3,2,3,2,1,5,2,1,6,
1,6,2,2,1,7,1,1,2,3,2,1,3,2,1,2,7,3,1,2,3,4,4,1,6,
4,1,2,4,2,2,1,6,4,1,1,1,2,4,2,1,4,1,1,1,3,3,2,2,1,

Again, the question is asked, what is the smallest number requiring k steps for the iterated function h(n) to reach 0? Below is a table of these numbers for k <= 12.

Number of steps	Smallest number
1	1
2	2
3	3
4	12
5	31
6	23
7	56
8	102
9	193
10	257
11	570
12	1129

The interesting thing to notice in the above table is that due to the reversal of lpd(n) and Lpf(n), the "smallest number" sequence above is not monotically increasing, i.e. 31 is the smallest number which takes 5 steps to reach 0, while 23 is the smallest number which takes 6 steps to reach zero. Also, note that there are seven primes and five non-primes in the above sequence. So one class is not predominating this sequence as in the others, at least for the first twelve values.

Open problem: What is the value requiring 13 steps?

We conjecture that this sequence is finite (although not necessarily at k=12).

(D) Now we will use the function in section B, except we will reverse its divisor/factor elements. That is, we will use:

i(n) = abs(reverse(Lcdd(n))-reverse(Lpf(n)))

and observe what happens when it is iterated. First notice that i(n) is not <= n for all n:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
i(n)	1	1	1	0	1	0	1	2	3	0	10	3	10	0	4	6	10	6	10	4	34	0

Number of steps it takes i(n) to reach one or zero for n=1 to 100:

0,1,1,1,1,1,1,2,2,1,2,2,2,1,2,2,2,2,2,2,2,1,2,3,2,
1,3,2,2,2,2,3,3,1,2,3,2,1,3,2,2,2,2,3,2,1,2,4,3,3,
3,3,2,5,4,2,3,1,2,2,2,1,3,3,2,2,2,3,4,2,2,3,2,1,1,
3,5,2,2,1,3,1,2,3,3,1,2,4,2,4,3,4,3,1,4,4,2,3,5,1,

Table for k <= 19 of smallest number requiring k steps for the iterated function i(n) to reach one or zero:

Number of steps	Smallest number
1	2
2	8
3	24
4	48
5	54
6	176
7	215
8	161
9	287
10	650
11	512
12	609
13	432
14	455
15	749
16	774
17	2650
18	2945
19	2997

Again this sequence is not monotonically increasing. There are no primes except 2 in the "smallest number" sequence above since for any prime p, i(p) will always give a power of 10; this follows from the definition of our function. To see this, let's take the prime 569 as an example. Lcdd(569)= 568 and Lpf(569)=569. When we reverse and subtract we get 965-865=100. So, for any prime p, i(p)=10^l(p)-1, where l(x) is the number of digits of x. And for all p > 7, i(p) will take 2 steps to reach 0. We conjecture that this sequence is finite (although not necessarily at k=19).

CONCLUSION

We have introduced four new functions having to do with prime factors and divisors of integers, made four conjectures regarding the finiteness of sequences involving Smarandache Iterations of the First Kind upon these functions, as well as giving some open problems. Our motivation for using prime factors and divisors in the functions is that other iterations have been explored with operations of multiplication [4] and addition of digits, along with some of the more common number theoretic functions [5], and thus we thought it would be interesting to investigate the underlying structure of integers through some unusual functions involving divisibility concepts when performing Smarandache Iterations of the First Kind.

On a computer related note, we realize that an interpreted algebra package such as PARI/GP, which the author used when preparing this paper, is not the best way to investigate the open problems given. A better way would be to write much faster programs in C or a similar programming language.

In closing, we suggest one more idea that we have not yet explored. The author thinks it would be very interesting to iterate the function j(n)=Lpf(n)-Ndcd(n), where Lpf(n) is the largest prime factor of n and Ndcd(n) is the number of distinct differences between consecutive divisors of n, when ordered by size (A060682) [1]. Let us know your results regarding iteration of this function!

REFERENCES

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