Smarandache Non-Associative (SNA-) rings

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In this paper we introduce the concept of Smarandache non-associative rings, which we shortly denote as SNA-rings as derived from the general definition of a Smarandache Structure (i.e., a set A embedded with a week structure W such that a proper subset B in A is embedded with a stronger structure S). Till date the concept of SNA-rings are not studied or introduced in the Smarandache algebraic literature. The only non-associative structures found in Smarandache algebraic notions so far are Smarandache groupoids and Smarandache loops introduced in 2001 and 2002. But they are algebraic structures with only a single binary operation defined on them that is non-associative. But SNA-rings are non-associative structures on which are defined two binary operations one associative and other being non-associative and addition distributes over multiplication both from the right and left. Further to understand the concept of SNA-rings one should be well versed with the concept of group rings, semigroup rings, loop rings and groupoid rings. The notion of groupoid rings is new and has been introduced in this paper. This concept of groupoid rings can alone provide examples of SNA-rings without unit since all other rings happens to be either associative or non-associative rings with unit. We define SNA subrings, SNA ideals, SNA Moufang rings, SNA Bol rings, SNA commutative rings, SNA non-commutative rings and SNA alternative rings. Examples are given of each of these structures and some open problems are suggested at the end.

Keywords: Non-associative ring, groupoid ring, group ring, loop ring, semigroup ring, SNA-rings SNA subrings, SNA ideals, SNA Moufang rings, SNA Bol rings, SNA commutative rings, SNA non-commutative rings and SNA alternative rings.

This paper has 5 sections. In the first section we just recall briefly the definition of non-associative rings and groupoid rings. In section 2 we define SNA-rings and give examples. Section 3 is devoted to the study of the two substructures of the SNA-rings and obtaining some interesting results about them. The study of rings satisfying identities happens to be a very important concept in the case of non-associative structures. So in this section we introduce several identities on SNA-rings and study them. The final section is devoted to some research problems, which alone can attract students and researchers towards the subject.

1. Preliminaries

This section is completely devoted to recollection of some definitions and results so as to make this paper self-contained.

Definition : A ring (R, +, o) is said to be a non-associative ring if (R, +) is an additive abelian group, (R, o) is a non-associative semigroup (that is the binary operation o on R is non-associative) such that the distributive laws

$a \circ (b + c) = a \circ b + a \circ c$ and $(a + b) \circ c = a \circ c + b \circ c$ for all $a, b, c \in R$ are satisfied.

Definition : Let R be a commutative ring with one. G any group (S any semigroup with unit) RG (RS the semigroup ring of the semigroup S over the ring R) the group ring of the group G over the ring R consists of finite formal sums of the form $\sum_{i=1}^{n} \alpha_i g_i$ $(n < \infty)$ that is i runs over a finite number where $\alpha_i \in R$ and $g_i \in G$ $(g_i \in S)$ satisfying the following conditions

i. $\sum_{i=1}^{n} \alpha_i m_i = \sum_{i=1}^{n} \beta_i m_i \Leftrightarrow \alpha_i = \beta_i$ for $i = 1, 2, \ldots, n$.

ii. $\sum_{i=1}^{n} \alpha_i m_i + \sum_{i=1}^{n} \beta_i m_i \Leftrightarrow \sum_{i=1}^{n} (\alpha_i + \beta_i) m_i$

iii. $\left(\sum_{i=1}^{n} \alpha_i m_i\right)\left(\sum_{i=1}^{n} \beta_i m_i\right) = \sum \gamma_k m_k$, $m_k = m_i m_j$ where $\gamma_k = \Sigma \alpha_i \beta_j$

iv. $r_i m_i = m_i r_i$ for all $r_i \in R$ and $m_i \in G$ $(m_i \in S)$.

v. $r \sum_{i=1}^{n} r_i m_i = \sum_{i=1}^{n} (r r_i) m_i$ for all $r \in R$ and $\Sigma r_i m_i \in RG$. RG is an associative ring with $0 \in R$ acts as its additive identity. Since $I \in R$ we have $G = I.G \subseteq RG$ and $R.e = R \subseteq RG$ where e is the identity element of G.

For more about group rings and semigroup rings please refer [4, 7, 10]. If we replace the group G in the above definition by a loop L we get RL the loop ring which will satisfy all the 5 conditions (i) to (v) given in definition. But RL will only be a non-associative ring as $I \in R$ and $e \in L$ we have $R \subseteq RL$ and $L \subseteq RL$. Any loop ring RL is an example of a non-associative ring with unit. For more about loop rings please refer [1, 3, 6, 8, 9] and about loops and groupoids refer [1, 2]. Now we define groupoid rings. Groupoid rings though not found in any literature to the best of our knowledge can be defined for any commutative ring R with identity 1. For G any groupoid the groupoid ring RG is the groupoid G over the ring R consists of all finite formal sums of the form $\sum_i r_i g_i$ (i running over finite integer) $r_i \in R$ and $g_i \in G$ satisfying the conditions i to v given in the definition of group rings. But a groupoid ring is a non-associative ring as G is non-associative. Clearly $IG \subset RG$ but $R \not\subset RG$ in general for the groupoid G may or may not contain the identity element in it. Thus only when the groupoid G has the identity element 1 we say the groupoid ring RG to be a non-associative ring with unit. Here we give examples of a non-associative ring without unit.

Example 1.1: Let Z be the ring of integers and L be a loop given by the following table:

*	1	a_1	a_2	a_3	a_4	a_5
1	1	a_1	a_2	a_3	a_4	a_5
a_1	a_1	1	a_3	a_5	a_2	a_4
a_2	a_2	a_5	1	a_4	a_1	a_3
a_3	a_3	a_4	a_1	1	a_5	a_2
a_4	a_4	a_3	a_5	a_2	1	a_1
a_5	a_5	a_2	a_4	a_1	a_3	e

Clearly the loop ring ZL is a non-associative ring with unit.

Example 1.2: Let Z be the ring of integers and (G, *) be the groupoid given by the following table:

*	a_0	a_1	a_2	a_3	a_4
a_0	a_0	a_2	a_4	a_1	a_3
a_1	a_1	a_3	a_0	a_2	a_4
a_2	a_2	a_4	a_1	a_3	a_0
a_3	a_3	a_0	a_2	a_4	a_1
a_4	a_4	a_1	a_3	a_0	a_2

Clearly (G, *) is a groupoid and (G, *) has no identity element. The groupoid ring ZG is a non-associative ring without unit element.

For more about groupoids, loops, loop ring, group ring, semigroup rings, please refer [1-10].

Result: All loop rings RL of a loop L over the ring R are non-associative rings with unit. The smallest non-associative ring without unit is of order 8 given by the following example.

Example 1.3: Let $Z_2 = \{0, 1\}$ be the prime field of characteristic 2. (G, *) be a groupoid of order 3 given by the following table:

*	g_1	g_2	g_3
g_1	g_1	g_2	g_4
g_2	g_4	g_1	g_2
g_3	g_2	g_4	g_1

Z_2G is the groupoid ring having only 8 elements given by $\{0, g_1, g_2, g_3, g_1 + g_2, g_2 + g_3, g_1 + g_3, g_1 + g_2 + g_3\}$. Clearly, Z_2G is a non-associative ring without unit. This is the smallest non-associative ring without unit known to us.

2. SNA-rings with Examples

Here we introduce the notion of SNA-rings and illustrate them with examples.

Definition 2.1: Let S be a non-associative ring. S is said to be a SNA-ring if S contains a proper subset P such that P is an associative ring under the operations of S.

Example 2.1: Let Z be the ring of integers and L be the loop given by the following table. ZL the loop ring of the loop L over the ring Z is a SNA-ring.

*	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
e	e	a_1	a_2	a_3	a_4	a_5	a_6	a_7
a_1	a_1	e	a_5	a_2	a_6	a_3	a_7	a_4
a_2	a_2	a_5	e	a_6	a_3	a_7	a_4	a_1
a_3	a_3	a_2	a_6	e	a_7	a_4	a_1	a_5
a_4	a_4	a_6	a_3	a_7	e	a_1	a_5	a_2
a_5	a_5	a_3	a_7	a_4	a_1	e	a_2	a_6
a_6	a_6	a_7	a_4	a_1	a_5	a_2	e	a_3
a_7	a_7	a_4	a_1	a_5	a_2	a_6	a_3	e

For $Z \,.\, e = Z \subseteq ZL$. Z is a proper subset of ZL, which is an associative ring. Further if $H_i = \langle e, a_i \rangle$ is the cyclic group generated by a_i; for i =1, 2, 3, …, 7. Clearly $ZH_i \subseteq ZL$ is the group ring of the group H_i over Z which is a proper subset of ZL. So ZL is a SNA-ring leading as to enunciate the following interesting theorem.

Theorem 2.2: Let L be a loop and R any ring. The loop ring RL is always a SNA-ring.

Proof: Clearly by the very definition of the loop ring RL we have $RI \subseteq RL$ so the ring R serves a non-empty proper subset, which is an associative ring. Hence the claim.

Example 2.2: Let R be the reals, (G, *) be the groupoid given by the following table:

*	0	1	2	3
0	0	3	2	1
1	2	1	0	3
2	0	3	2	1
3	2	1	0	3

RG is a non-associative ring which is a SNA-ring as R ⟨2⟩ is an associative ring which is a proper subset of RG as this SNA-ring has no unit element. Thus it is a Smarandache non-associative ring without unit When we take $0 \in G$ we assume $r0 \neq 0$ for all non-zero $r \in R$ and $0g = 0$ for all $g \in G$.

Example 2.3: Let Z be the ring of integers. (G, *) be a groupoid given by the following table:

*	0	1	2	3	4	5
0	0	4	2	0	4	2
1	2	0	4	2	0	4
2	4	2	0	4	2	0
3	0	4	2	0	4	2
4	2	0	4	2	0	4
5	4	2	0	4	2	0

Consider the groupoid ring ZG, this has no identity but ZG is a non-associative ring, which has a proper subset ZH, where H = {0, 3} is a semigroup so ZH is an associative ring. Thus ZG is a SNA-ring.

Example 2.4: Let Q be the field of rationals. (G, *) be the groupoid with unit element e given by the following table:

*	e	0	1	2	3	4
e	e	0	1	2	3	4
0	0	e	1	2	3	4
1	1	2	e	4	0	1
2	2	4	0	e	2	3
3	3	1	2	3	e	0
4	4	3	4	0	1	e

Clearly the groupoid ring QG is a SNA-ring $Q \,.\, e = Q \subseteq QG$ where Q is the associative ring. Further QG is a SNA-ring with unit. Now in view of these examples we obtain the following results.

<u>**Theorem 2.3**</u>: Let R be any ring and G a groupoid with identity. Then the groupoid ring RG is a SNA-ring.

Proof: Obvious from the fact that identity element exists in G so $R \,.\, I = R \subseteq RG$ so R serves as the associative ring to make RG a SNA ring with unit.

<u>**Theorem 2.4**</u>: Let R be a ring if G is a Smarandache groupoid then the groupoid ring RG is a SNA-ring.

Proof: Clearly the groupoid ring RG is a non-associative ring. Given G is a Smarandache groupoid; so by definition of Smarandache groupoid G contains non-empty subset P of G such that P is a semigroup. RP is a semigroup ring of the semigroup P over the ring R, so that RP is an associative ring, which is a proper subset of RG. Thus RG is a SNA-ring.

3. Substructures of SNA-rings

In this section we introduce the two substructures viz. SNA subrings and SNA ideals.

Definition 3.1: Let R be a non-associative ring. A non-empty subset S of R is said to be a SNA subring of R if S contains a proper subset P such that P is an associative ring under the operations of R.

Now we have got two nice results about these SNA subrings, which are enunciated as theorem.

Theorem 3.2: Let R be a non-associative ring; if R has a SNA subring then R is a SNA subring.

Proof: Given R is a non-associative ring such that R contains a proper subset S which is a SNA subring that is S contains a proper subset P which is an associative ring. Now $P \subset S$ and $S \subset R$ so $P \subset R$ that is R has a proper subset P that is an associative ring. Hence R is a SNA-ring.

To prove the next theorem we consider the following example.

Example 3.1: Let Z be the ring of integers (G, *) be the groupoid given in example 2.3. Clearly the groupoid ring ZG is a non-associative ring. Now consider the subset P = {0, 2, 4} P is a sub groupoid of G so ZP is also a groupoid ring ,which is non-associative and ZP is a subring of ZG. Clearly ZP is not an associative subring. So in view of theorem 3.2 we can say if R is a SNA-ring and has a subring which is not a SNA subring of R.

This leads us to the following theorem.

Theorem 3.3: Let R be a SNA-ring. Every subring of R need not in general be a SNA subring of R.

Proof: From example 3.1 we see that ZH where H is generated by (0, 2, 4) is a subring of R as it has no proper subset, which is a non-associative ring. So ZH is a subring, which is not a SNA subring of R.

Now we proceed on to define SNA ideal.

Definition 3.4: Let R be any non-associative ring. A proper subset I of R is said to be a SNA right/left ideal of R if

1. I is a SNA subring of R; say J ⊂ I, J is a proper subset of I which is an associative subring under the operations of R.
2. For all i ∈ I and j ∈ J we have either ij or ji is in J

If I is simultaneously both a SNA right ideal and SNA left ideal then we say I is a SNA ideal of R.

Example 3.2. Let Z be the ring of integers (G, *) be a groupoid of order 8 given by the following table:

*	0	1	2	3	4	5	6	7
0	0	6	4	2	0	6	4	2
1	3	1	7	5	3	1	7	5
2	6	4	2	0	6	4	2	0
3	1	7	5	3	1	7	5	3
4	4	2	0	6	4	2	0	6
5	7	5	3	1	7	5	3	1
6	2	0	6	4	2	0	6	4
7	5	3	1	7	5	3	1	7

Clearly ZG is a SNA-ring as H = {2} is a semigroup. The semigroup ring ZH is a non-empty proper subset, which is an associative ring. Clearly I = Z ⟨0, 2, 4, 6⟩ is a SNA ideal of ZG. It is easily verified that I = Z⟨0, 2, 4, 6⟩ is not an ideal of ZG. Similarly we see I_1 = Z⟨1, 3, 5, 7⟩ is also a SNA ideal of ZG, which is not an ideal of ZG. Consequent of this example and the definition of SNA ideals we have following two theorems.

Theorem 3.5. Let R be any non-associative ring. If R has a SNA ideal then R is a SNA-ring.

Proof: Obvious from the fact that if R has a SNA ideal say I then we have proper subset J ⊂ I such that J is a SNA subring of R. So by theorem 3.3 R is a SNA-ring.

Theorem 3.6: Let R be any non-associative ring. I be a SNA ideal of R. Then I in general need not be an ideal of R.

Proof: By an example. Consider the non-associative ring given in example 3.2. Clearly Z⟨0, 2, 4, 6⟩ is a SNA ideal of ZG but Z⟨0, 2, 4, 6⟩ is not an ideal of ZG as 3[Z⟨0, 2, 4, 6⟩] = Z⟨1, 3, 5, 7⟩. Clearly Z⟨0, 2, 4, 6⟩ ≠ Z⟨1, 3, 5, 7⟩ in fact they are disjoint sets. Hence the claim.

Example 3.3: Let Z be the ring of integers. (G, *) be as given in example 2.3. Clearly Z⟨0, 2, 4⟩ is an ideal of ZG but Z⟨0, 2, 4⟩ is not a SNA ideal of ZG as Z⟨0, 2, 4⟩ has no proper subset P such that P is an associative subring of Z⟨0, 2, 4⟩. Hence the claim.

4. SNA-rings satisfying certain identities

In this section we define SNA-rings satisfying certain classical identities like Bol, Moufang etc. and obtain some interesting results relating to the loop rings of the loop and groupoid rings of the groupoid. We give examples of them to make it explicit.

Definition 4.1: Let R be a non-associative ring we say R is a SNA Moufang ring if R contains a subring S where S is a SNA subring and for all x, y, z in S we have
(x * y) * (z * x) = (x * (y * z)) * x, that is the Moufang identity to be true in S.

Examples 4.1. Let Z be the ring of integers and let (L, .) be the loop given by the following example:

o	e	g_1	g_2	g_3	g_4	g_5
e	e	g_1	g_2	g_3	g_4	g_5
g_1	g_1	e	g_3	g_5	g_2	g_4
g_2	g_2	g_5	e	g_4	g_1	g_3
g_3	g_3	g_4	g_1	e	g_5	g_2
g_4	g_4	g_3	g_5	g_2	e	g_1
g_5	g_5	g_2	g_4	g_1	g_3	e

Clearly L is not a Moufang loop. Consider the loop ring ZL. ZL is a non-associative which is a SNA-ring. Clearly L is not a Moufang loop. But ZL is a SNA-Moufang ring as Z⟨e,g_1⟩ is a proper subset of ZL such that Z ⊆ Z⟨e, g_1⟩ is an associative subring of Z⟨e,g_1⟩. Now it is easily verified Z⟨e, g_1⟩ satisfies the Moufang identity for every x, y, z ∈ Z⟨e, g_1⟩.

Example 4.2: Let Z be the ring of integers (G, *) be the groupoid given by the following table:

*	0	1	2	3	4	5
0	0	4	2	0	4	2
1	3	1	5	3	1	5
2	0	4	2	0	4	2
3	3	1	5	3	1	5
4	0	4	2	0	4	2
5	3	1	5	3	1	5

ZG is the groupoid ring of G over Z. Clearly, every subring of ZG satisfies Moufang identity as every element of ZG satisfies Moufang identity, in fact ZG is a non-associative ring, which satisfies Moufang identity so ZG is a SNA-ring. Here it has become important to say that one needs to define such rings as these rings have not been found any place in literature.

Definition 4.2. A non-associative ring R is said to be a Moufang ring if the Moufang identity, (x * y) * (z * x) = (x * (y * z))* x is satisfied for all x, y, z ∈ R.

In view of this we have the following interesting result.

Theorem 4.3: If R is a Moufang ring and if R is a SNA-ring Then R is a SNA Moufang ring.

Proof: By the very definition used in this paper.

Definition 4.4: Let R be a non-associative ring R is said to a Bol ring if R satisfies the Bol identity ((x * y) * z) *y = x * ((y * z) * y) for all x, y, z in R.

Trivially all associative rings satisfy Bol identity hence we take only non-associative rings.

Definition 4.5: Let R be a non-associative ring. R is a said to be a SNA Bol ring if R contains a subring S ⊂ R such that S is a SNA subring of R and we have the Bol identity ((x * y) * z) * y = x * ((y * z) * y) to be true for all x, y, z in S.

In view of this we have the following theorem.

Theorem 4.6: Let R be a non-associative ring, which is a Bol ring .If R, is also a SNA-ring then R is a SNA Bol ring.

Proof: Clear from the very definitions given in this paper.

Example 4.3: Let Z be the ring of integers, L be the loop given by the following table:

*	e	1	2	3	4	5	6	7
e	e	1	2	3	4	5	6	7
1	1	e	5	2	6	3	7	4
2	2	5	e	6	3	7	4	1
3	3	2	6	e	7	4	1	3
4	4	6	3	7	e	1	5	2
5	5	3	7	4	1	e	2	6
6	6	7	4	1	5	2	e	3
7	7	4	1	5	2	6	3	e

Clearly this loop is not a Bol loop so the loop ring ZL is not a Bol ring. But this loop ring ZL is a SNA Bol ring as Z ⊆ Z(e, 5) ⊂ ZL is a SNA Bol ring.

In view of this we have the following theorem.

Theorem 4.7: If R is a non-associative ring which is a SNA Bol ring R need not in general be a Bol ring.

Proof: Using the very definition and the example 4.3 we get the result.

Definition 4.8: Let R be any non-associative ring, R is said to be a right alternative ring if (xy) y = x (yy) for all x, y ∈ R. Similarly R is said to be left alternative ring if (xx) y = x (xy) for all x, y ∈ R. Finally we say R is an alternative ring if it is simultaneously both right alternative and left alternative.

Example 4.4: Let Z be the ring of integers and L be a loop given by the following table:

*	e	g_1	g_2	g_3	g_4	g_5
e	e	g_1	g_2	g_3	g_4	g_5
g_1	g_1	e	g_3	g_5	g_2	g_4
g_2	g_2	g_5	e	g_4	g_1	g_3
g_3	g_3	g_4	g_1	e	g_5	g_2
g_4	g_4	g_3	g_5	g_2	e	g_1
g_5	g_5	g_2	g_4	g_1	g_3	e

The loop ring ZL is a right alternative ring as the loop L itself a right alternative loop.

Example 4.5: Let Z be the ring of integers and L be a loop given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	5	4	3	2
2	2	3	e	1	5	4
3	3	5	4	e	2	1
4	4	2	1	5	e	3
5	5	4	3	2	1	e

Consider the loop ring ZL, it is easily verified that ZL is a left alternative ring as the loop L is left alternative. In view of this we have the following results, which will be stated after defining the concept of SNA alternative rings.

Definition 4.9: Let R be a ring, R is said to be SNA right alternative ring if R has a subring S such that S is a SNA subring of R and S is a right alternative ring that is (xy) y = x (yy) is true for all x, y ∈S. Similarly we define SNA left alternative ring. If R is simultaneously both SNA right alternative ring and SNA left alternative then we say R is a SNA alternative ring.

Example 4.6: Let Z be the ring of integers. (G, *) be the groupoid given by the following table:

*	0	1	2	3	4	5
0	0	3	0	3	0	3
1	4	1	4	1	4	1
2	2	5	2	5	2	5
3	3	0	3	0	3	0
4	4	1	4	1	4	1
5	2	5	2	5	2	5

The groupoid ring ZG is a SNA-ring. Further, we have ZG to be an alternative ring as well as a SNA alternative ring.

Definition 4.10: Let R be non-associative ring. R is said to be a SNA commutative ring if R has a subring S such that a proper subset P of S is a commutative associative ring with respect to the operations of R.

Note: Even if R is non-commutative, still R can be a SNA commutative ring. Further we see trivially all commutative non-associative rings R will be SNA commutative rings. We say R is a SNA non-commutative ring if R has no SNA commutative subring.

Example 4.7: Let Z be the ring of integers and L be a loop given by the following table:

*	e	1	2	3	4	5
e	e	1	2	3	4	5
1	1	e	3	5	2	4
2	2	5	e	4	1	3
3	3	4	1	e	5	2
4	4	3	5	2	e	1
5	5	2	4	1	3	e

The loop ring ZL is a non-associative ring. Clearly ZL is also a SNA commutative ring. As $Z \subset Z\,(e, 3) \subset ZL$. Z(e, 3) is a SNA subring of ZL, which has a proper subset Z, Z is an associative commutative subring of ZL. Thus we ZL is non commutative but ZL is a SNA commutative ring.

Example 4.8: Let Z be the ring of integers (G, *) be a groupoid given by the following table:

*	0	1	2	3	4	5
0	0	5	4	3	2	1
1	2	1	0	5	4	3
2	4	3	2	1	0	5
3	0	5	4	3	2	1
4	2	1	0	5	4	3
5	4	3	2	1	0	5

Consider the groupoid ring ZG. Clearly ZG is non-associative non-commutative ring. But ZG is a SNA commutative ring as $Z\langle 3\rangle \subseteq Z\langle 0, 3\rangle \subset ZG$. Clearly ZG is non-commutative but ZG is SNA commutative ring. Hence the claim.

5. Problems:

This section is completely devoted to some open problems some may be easy and some of them may be difficult.

Problem 1: Find the smallest non-associative ring. (By smallest we mean the number of elements in them that is order is the least that is we cannot find any other non-associative ring of lesser order than that).

Problem 2: Is the smallest non-associative ring a SNA-ring?

Problem 3: Find SNA-ring of least order.

Problem 4: Can on Z_n be defined two binary operations so that Z_n is non-associative ($n < \infty$)?

Problem 5: Find the smallest SNA-ring, which is a SNA Bol ring.

Problem 6: Does their exist SNA-rings other than the ones got from

1. loop rings,
2. groupoid rings

Problem 7: Find a SNA-ring R in which every ideal of R is a SNA ideal of R.

Problem 8: Find conditions on the ring R so that every subring of R is a SNA subring of R.

Problem 9: Characterize the SNA-rings R which has ideals but none of them are SNA ideals of R.

Problem 10: Characterize those ring R in a SNA-ring which has subrings but none of the subrings in R are SNA subrings of R.

References

[1] R. H. Bruck, Contributions to the theory of loops, Trans. Amer. Math. Soc., Vol. 60, 245 – 354, (1946).

[2] R. H. Bruck, A survey of binary systems, Springer Verlag, (1958).

[3] O. Chein and E. Goodaire, Loops whose loop rings are alternative, Communications in Algebra, Vol. 18(3), 659 – 668, (1990).

[4] D.S.Passman, The algebraic structure of group rings, Wiley-Interscience, (1977).

[5] W. B. Vasantha Kandasamy, On ordered groupoids and groupoid rings, J. of Maths. and Comp. Sci., Vol. 9, 145-147, (1996).

[6] W. B. Vasantha Kandasamy, Loop algebras of Moufang loops, Math. Forum, Vol.XI, 21-25, (1997).

[7] W. B. Vasantha Kandasamy, Semigroup rings which are Chinese rings, J. of Math. Research & Exposition, Vol.13, 375 – 376, (1993).

[8] W. B. Vasantha Kandasamy, On loops rings which are Marot loop rings, Ultra Scientist Phys. Sci., 126 – 128, (1996).

[9] W. B. Vasantha Kandasamy, On a new class of Jordan loops and their loop rings, J. Bihar Math. Soc. 71 – 75, (1999).

[10]. W. B. Vasantha Kandasamy, On semigroup rings which satisfy $(xy)^n = x.y$, J. of Bihar Math. Soc. , Vol. 14, 47 – 50, (1995).

[11]. W. B. Vasantha Kandasamy, Smarandache loops, www.gallup.unm.edu/~smarandache/Loops.pdf

[12]. W. B. Vasantha Kandasamy, Smarandache groupoids, www.gallup.unm.edu/~smarandache/Groupoids.pdf