SMARANDACHE NUMBER RELATED TRIANGLES

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ABSTRACT

Given a triangle in Euclidean geometry it is well known that there exist an infinity of triangles each of which is similar to the given one. In Section I we make certain observations on Smarandache numbers. This enables us to impose a constraint on the lenghts of the corresponding sides of similar triangles. In Section II we do this to see that infinite class of similar triangles reduces to a finite one. In Section III we disregard the similarity requirement. Finally, in Section IV we pose a set of open problems.

I SMARANDACHE NUMBERS: SOME OBSERVATIONS

Suppose a natural number n is given. The Smarandache number of n is the least number denoted by S(n) whith the following property: n divides S(n)! but not (S(n)-1)!. Below is a short table containing n and S(n) for $1 \le n \le 12$.

n	1	2	3	4	5	6	7	8	9	10	11	12
S(n)	1	2	3	4	5	3	7	4	6	5	11	4

A look at the above table shows that S(3) = S(6) = 3, S(4) = S(8) = S(12) = 4, ...

Let a natural number k be given. Then the equation S(x) = k cannot have an infinity of solutions x. This is because the largest solution is x=k!. This observation enables us to impose a restriction on the lengths of the corresponding sides of similar triangles. In the next section we shall see how to do this. Throughout this paper the triangles are assumed to have natural number side lengths. Also, the triangles are non degenerate.

II SMARANDACHE SIMILAR TRIANGLES

Let us denote by T(a,b,c) the triangle ABC with side lengths a, b, c. Then the two similar triangles T(a,b,c) and T'(a',b',c') are said to be Smarandache Similar if S(a) = S(a'), S(b) = S(b'), S(c) = S(c'). Trivially, a given triangle is Smarandache similar to itself. Non trivially the two Pytagorean triangles (right triangles with natural number side lengths) T(3,4,5) and T'(6,8,10)

are Smarandache similar because
$S(3) = S(6) = 3$, $S(4) = S(8) = 4$, $S(5) = S(10) = 5$.
However, the Similar triangles (3,4,5) and (9,12,15) are not Smarandache Similar because $S(3) = 3 \neq S(9)$ which is 6. In fact the class of Smarandache Similar triangles generated by T(3,4,5) contains just two: T(3,4,5) and T'(6,8,10) in view of the fact that the solution set of the equation $S(x) = 3$ consists of just two members $x = 3, 6$.

For another illustration let us determine the class of Smarandache Similar Triangles generated by the 60° triangle $T(a,b,c) = (5,7,8)$. The algorithm to do this is as follows: First we calculate $S(5) = 5$, $S(7) = 7$, $S(8) = 4$. Next we solve the equations $S(a') = 5$, $S(b') = 7$, $S(c') = 4$. Let us solve the last equation first.

$S(c') = 4 \rightarrow c' = 4, 8, 12, 24$.

Here the largest value $c' = 24 = 3c$. Hence we need not to solve the other two equations beyond the solutions $a' = 3a$, $b' = 3b$. This observation therefore gives us

$S(a') = 5 \rightarrow a' = 5, 10, 15$ and

$S(b') = 7 \rightarrow b' = 7, 14, 21$.

It is now clear that the class of Smarandache similar triangles contains just two members: (5,7,8) and (15,21,24).

III SMARANDACHE RELATED TRIANGLES

In this section we do not insist on the similarirty requirement that we had in Section II. Hence the definition: Given a triangle T(a,b,c) we say that a triangle T'(a',b',c') is Smarandache related to T if $S(a') = S(a)$, $S(b') = S(b)$, $S(c') = S(c)$. Note that the triangles T and T' may or may not be similar. As an illustration let us determine all the triangles that are Smarandache related to T(3,4,5). To do this we follow the same algorithm that we mentioned in Section II but we have to find all the solutions of the equations $S(a') = 3$, $S(b') = 4$, $S(c') = 5$. Therefore

$S(a') = 3 \rightarrow a' = 3, 6$;

$S(b') = 4 \rightarrow b' = 4, 8, 12, 24$;

$S(c') = 5 \rightarrow c' = 5, 10, 15, 20, 30, 40, 60, 120$.

This gives us the complete solution $(a',b',c') =$ (3,4,5); (3,8,10); (3,12,10); (6,4,5); (6,8,5); (6,8,10); (6,12,10); (6,12,15); (6,24,20).

IV CONCLUSION

In the present discussion I have used small natural numbers k so that the solution of the equations $S(x) = k$ can be easily determined. I do not know if this interesting converse problem of determining all natural numbers x for given k of the Smarandache equation $S(x)=k$ has been disscused by someone already. In case if this has not been already considered, I invite the reader to devise efficient methods to solve the preceding equation. We conclude this section by posing the following open problems to the reader.

(A) Are there two distinct dissimilar Phytagorean triangles that are Smarandache related? i.e. both $T(a,b,c)$ and $T'(a',b',c')$ are Pythagorean such that $S(a') = S(a)$, $S(b') = S(b)$, $S(c') = S(c)$ but T and T' are *not* similar.

(B) Are there two distinct and dissimilar 60° triangles (120° triangles) that are Smarandache related?

(c) Given a triangle $T(a,b,c)$. Is it possible to give either an exact formula or an uper bound for the total number of triangles (without actually determining all of them) that are Smarandache related to T?

(D) Consider other ways of relating two triangles in the Smarandache number sense. For example, are there two triplets of natural numbers (α,β,γ) and (α',β',γ') such that $\alpha + \beta + \gamma = \alpha' + \beta' + \gamma' = 180$ and $S(\alpha) = S(\alpha')$, $S(\beta) = S(\beta')$, $S(\gamma) = S(\gamma')$.
If such distinct triplets exist the two triangles would be Smarandache related via their angles. Of course in this relationship the side lengths of the triangles may not be natural numbers.