SMARANDACHE RECIPROCAL PARTITION OF UNITY SETS AND SEQUENCES

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ABSTRACT: Expression of unity as the sum of the reciprocals of natural numbers is explored . And in this connection Smarandache Reciprocal partition of unity sets and sequences are defined. Some results and Inequalities are derived and a few open problems are proposed.

DISCUSSION:

Define Smarandache Repeatable Reciprocal partition of unity set as follows:

$$SRRPS(n) = \{ x \mid x = (a_1, a_2, \ldots, a_n) \text{ where } \sum_{r=1}^{n} (1/a_r) = 1.\}$$

$f_{RP}(n)$ = order of the set SRRPS(n).

We have

SRRPS(1) = { (1) } , $f_{RP}(1) = 1$.

SRRPS(2) = { (2,2) } , $f_{RP}(2) = 1$.

SRRPS(3) = { (3,3,3),(2,3,6), (2,4,4) } , $f_{RP}(3) = 3$.,1 = 1/2 + 1/3 + 1/6 etc.

SRRPS(4) = { (4,4,4,4), (2,4,6,12), (2,3,7,42), (2,4,5,20),

(2,6,6,6),(2,4,8,8,),(2,3,12,12), (4,4,3,6) , (3,3,6,6), (2,3,10,15) }

$f_{RP}(4) = 10$.

SMARANDACHE REPEATABLE RECIPROCAL PARTITION OF UNITY SEQUENCE is defined as

1 , 1 , 3 , 10 . . .

where the n^{th} term = $f_{RP}(n)$.

Define **SMARANDACHE DISTINCT RECIPROCAL PARTITION OF UNITY SET**

as follows

SDRPS(n) = { x | x = (a_1, a_2, . . ., a_n) where $\sum_{r=1}^{n} (1/a_r) = 1$ and $a_i = a_j \Leftrightarrow i = j$}

$f_{DP}(n)$ = order of SDRPS(n).

SDRPS(1) = { (1) } , $f_{DP}(1) = 1$.

SDRPS(2) = { } , $f_{RP}(2) = 0$.

SDRPS(3) = { (2,3,6) } , $f_{DP}(3) = 1$.

SRRPS(4) = {(2,4,6,12), (2,3,7,42), (2,4,5,20),(2,3,10,15)}

$f_{DP}(4) = 4$.

Smarandache Distinct Reciprocal partition of unity sequences

defined as follows

1 , 0 , 1 , 4 , 12 . . .

the n^{th} term is $f_{DP}(n)$.

Following Inequality regarding the function $f_{DP}(n)$ has been

established.

THEOREM(1.1)

$$f_{DP}(n) \geq \sum_{k=3}^{n-1} f_{DP}(k) + (n^2 - 5n + 8)/2 \quad , n > 3 \qquad \text{----------(1.1)}$$

This inequality will be established in two steps.

Proposition (1.A)

For every n there exists a set of n natural numbers sum of whose reciprocals is 1.

Proof: This will be proved by induction. Let the proposition be true for $n = r$.

Let $a_1 < a_2 < a_3 < \ldots < a_{n-1} < a_n = k$ be r distinct natural numbers such that

$1/a_1 + 1/a_2 + 1/a_3 + \ldots + 1/a_r = 1$

We have , $1/k = 1/(k+1) + 1/(k(k+1))$, which gives us a set of r+1 distinct numbers $a_1 < a_2 < a_3 < \ldots < a_{r-1} < k+1 < k(k+1)$, sum of whose reciprocals is 1.

$P(r) \Rightarrow P(r+1)$, and as P(3) is true i.e. $1/2 + 1/3 + 1/6 = 1$,

The proposition is true for all n.

This completes the proof of proposition (1.A).

Note: If $a_1, a_2, a_3, \ldots a_{n-1}$ are n-1 distinct natural numbers given by

$a_1 = 2.$
$a_2 = a_1 + 1.$
$a_3 = a_1a_2 + 1$
.
.
.
$a_t = a_1a_2a_3\ldots a_{t-1} + 1. \quad = a_{t-1}(a_{t-1} - 1) + 1$
.
.
.
$a_{n-2} = a_1a_2a_3\ldots a_{n-3} + 1$
$a_{n-1} = a_1a_2a_3\ldots a_{n-2}$

then these numbers form a set of (n - 1) distinct natural numbers such that

$$\sum_{t=1}^{n-1} 1/a_t = 1.$$

we have $a_t = a_{t-1}(a_{t-1} - 1) + 1$ except when $t = n - 1$ in which

case

$a_{n-1} = a_{n-2}(a_{n-2} - 1)$

Let the above set be called **Principle Reciprocal Partition**.

*** It can easily be proved in the above set that

$a_{2t} \equiv 3 \mod(10)$ and $a_{2t+1} \equiv 7 \mod(10)$ for $t \geq 1$.

Consider the principle reciprocal partition for n-1 numbers . Each a_t contributes one to $f_{DP}(n)$ if broken into $a_t + 1$, $a_t(a_t + 1)$ except for $t = 1$. (as 2, if broken into 3 and 6, to give 1/2 =1/3 + 1/6, the number 3 is repeated and the condition of all distinct number is not fulfilled). There is a contribution of n - 2 from the principle set to $f_{DP}(n)$. The remaining $f_{DP}(n-1) -1$ members (excluding the principle partition) of SDRPS(n-1) would contribute at least one each to $f_{DP}(n)$ (breaking the largest number in each such set into two parts) . The contribution to $f_{DP}(n)$ thus is at least

$n-2 + f_{DP}(n-1) - 1 = f_{DP}(n-1) + n - 3$

$$\mathbf{f_{DP}(n) \geq f_{DP}(n-1) + n - 3} \qquad \text{----------}(1.2)$$

Also for each member $(b_1, b_2, \ldots, b_{n-1})$ of SDRPS(n-1) there exists a member of SDRPS(n) i.e. $(2, 2b_1, 2b_2, \ldots, 2b_{n-1})$ as we can see that

$1 = (1/2)(1 + 1/b_1 + 1/b_2 + \ldots + 1/b_{n-1}) = 1/2 + 1/2b_1 + \ldots + 1/2b_{n-1}$.

In this way there is a contribution of $f_{DP}(n-1)$ to $f_{DP}(n)$. -------(1.3)

Taking into account all these contributions to $f_{DP}(n)$ we get

$$f_{DP}(n) \geq f_{DP}(n-1) + n - 3 + f_{DP}(n-1)$$

$$f_{DP}(n) \geq 2f_{DP}(n-1) + n - 3$$

$$f_{DP}(n) - f_{DP}(n-1) \geq f_{DP}(n-1) + n - 3 \quad \text{-------------(1.4)}$$

from (4) by replacing n by n-1 , n-2 , etc. we get

$$f_{DP}(n-1) - f_{DP}(n-2) \geq f_{DP}(n-2) + n - 4$$

$$f_{DP}(n-2) - f_{DP}(n-3) \geq f_{DP}(n-3) + n - 5$$

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$$f_{DP}(4) - f_{DP}(3) \geq f_{DP}(3) + 1$$

summing up all the above inequalities we get

$$f_{DP}(n) - f_{DP}(3) \geq \sum_{k=3}^{n-1} f_{DP}(k) + \sum_{r=1}^{n-1} r$$

$$f_{DP}(n) \geq \sum_{k=3}^{n-1} f_{DP}(k) + (n-3)(n-2)/2 + 1$$

$$f_{DP}(n) \geq \sum_{k=3}^{n-1} f_{DP}(k) + (n^2 - 5n + 8)/2 \quad , \quad n > 3$$

Remarks : Readers can come up with stronger results as in my opinion the order of $f_{DP}(n)$ should be much more than what has been arrived at. This will be clear from the following theorem.

THEOREM(1.2):

If m is a member of an element of SRRPS(n) say,

$(a_1, a_2, a_3, \ldots, a_n)$.We have $a_k = m$ for some k and $\sum_{k=1}^{n} 1/a_k = 1$.

then m contributes $[\{d(m) + 1\}/2]$ elements to SRRPS(n+1), where the symbol [ ] stands for integer value and d(m) is the number of divisors of m.

Proof: For each divisor d of m there corresponds another divisor $m/d = d'$.

Case-I: m is not a perfect square . Then d(m) is even and there are d(m)/2 pairs of the type (d,d') such that $dd' = m$.

Consider the following identity

$1/(p.q) = 1/(p(p+q)) + 1/(q(p+q))$ ------------(1.5)

for each divisor pair (d,d') of m we have the following breakup

$1/(d.d') = 1/(d(d+d')) + 1/(d'(d+d'))$

Hence the contribution of m to SRRPS(n+1) is d(m)/2. As d(m) is even $d(m)/2 = [\{d(m)+1\}/2]$ Also.

Case-II m is a perfect square. In this case d(m) is odd and there is a divisor pair $d=d' = m^{1/2}$. This will contribute one to SRRPS(n+1) .The remaining {d(m) -1}/2 pairs of distinct divisors will contribute as many i.e. say ({d(m) -1}/2) .Hence the total contribution in this case would be

$\{d(m) -1\}/2 + 1 = \{d(m) +1\}/2 =[\{d(m) +1\}/2]$

Hence m contributes [{d(m) + 1}/2] elements to SRRPS(n+1)

This completes the proof.

Remarks:(1) The total contribution to SRRPS(n+1) by any element of SRRPS(n) is $\Sigma [\{d(a_k) + 1\}/2]$ ---------(1.6),

where each a_k is considered only once irrespective of its' repeated occurrence.

(2) In case of SDRPS(n+1) , the contribution by an element of SDRPS(n) is given by

$$\sum_{k=1}^{n} [\{d(a_k)\}/2] \qquad \text{-----------(1.7)}$$

because the divisor pair $d =d'= a_k^{\ 1/2}$ does not contribute.

Hence the total contribution of SDRP(n) to generate SDRPS(n+1) is the summation over all the elements of SDRPS(n) .

$$\sum_{f_{DP}(n)} \{ \sum_{k=1}^{n} [\{d(a_k)\}/2] \} \qquad \text{----------- (1.8)}$$

Generalizing the above approach.

The readers can further extend this work by considering the following identity

$$\frac{1}{pqr} = \frac{1}{pq(p+q+r)} + \frac{1}{qr(p+q+r)} + \frac{1}{rp(p+q+r)} \qquad \text{----(1.9)}$$

which also suggests

$$\frac{1}{b_1 b_2 \ldots b_r} = \sum_{k=1}^{r} \left\{\left(\prod_{t=1, t\neq k}^{r} b_t\right)\left(\sum_{s=1}^{r} b_s\right)\right\}^{-1} \qquad \text{-------(1.10)}$$

The above identity can easily be established by just summing up the right hand member.

From (1.10) , the contribution of the elements of SDRPS(n) to SDRPS(n+r) can be evaluated if an answer to following tedious querries could be found.

OPEN PROBLEMS:

(1) In how many ways a number can be expressed as the product of 3 of its divisors?

(2) In general in how many ways a number can be expressed as the product of r of its' divisors?

(3) Finally in how many ways a number can be expressed as the product of its divisors?

An attempt to get the answers to the above querries leads to the need of the generalization of the theory of partition function.

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