SMARANDACHE RELATIONSHIPS AND SUBSEQUENCES*

M. Bencze
2212 Sacele, Grasov, Romania

ABSTRACT

Some Smarandache relationships between the terms of a given sequence are studied in the first paragraph. In the second paragraph, are studied Smarandache subsequences (whose terms have the same property as the initial sequence). In the third paragraph are studied the Smarandache magic squares and cubes of order n and some conjectures in number theory.

Key Words: Smarandache p-q relationships, Smarandache p-q-<>-subsequence, Smarandache type subsequences, Smarandache type partition, Smarandache type definitions, Smarandache type conjectures in number theory.

1) Smarandache Relationships

Let $\{ a_n \}$, $n \geq 1$ be a sequence of numbers and p, q integers ≥ 1. Then we say that the terms

$$a_{k+1}, a_{k+2}, \dots, a_{k+p}, a_{k+p+1}, a_{k+p+2}, \dots, a_{k+p+q}$$

verify a *Smarandache p-q relationship* if

$$a_{k+1} <> a_{k+2} <> \dots <> a_{k+p} = a_{k+p+1} <> a_{k+p+2} <> \dots <> a_{k+p+q}$$

where "<>" may be any arithmetic or algebraic or analytic operation (generally a binary law on $\{ a_1, a_2, a_3, \dots \}$).

If this relationship is verified for any $k \geq 1$ (i.e. by all terms of the sequence), then

$\{ a_n \}$, $n \geq 1$ is called a *Smarandache p-q-<> sequence*

where "<>" is replaced by "additive" if $<> = +$, "multiplicative" if $<> = *$, etc. [according to the operation (<>) used].

As a particular case, we can easily see that Fibonacci/Lucas sequence

$(a_n + a_{n+1} = a_{n+2})$, for $n \geq 1$

is a Smarandache 2-1 additive sequence.

A Tribonacci sequence $(a_n + a_{n+1} + a_{n+2} = a_{n+3})$, $n \geq 1$ is a Smarandache 3-1 additive sequence. Etc.

Now, if we consider the sequence of Smarandache numbers,

1, 2, 3, 4, 5, 3, 7,4 , 6, 5, 11, 4, 13, 7, 5, 6, 17, . . . , i.e. for each n the smallest number S(n) such that S(n)! is divisible by n [See(1)] (the values of the Smarandache Function), it raises the questions:

(a) How many quadruplets verify a Smarandache 2-2 additive relationship i.e.

$S(n+1) + S(n+2) = S(n+3) + S(n+4)$?

I found: $S(6) + S(7) = S(8) + S(9)$, $3 + 7 = 4 + 6$;

$S(7) + S(8) = S(9) + S(10)$, $7 + 4 = 6 + 5$;

$S(28) + S(29) = S(30) + S(31)$, $7 + 29 = 5 + 31$.

But, what about others? I am not able to tell you if there exist a finite or infinite number (?)

(b) How many quadruplets verify a Smarandache 2-2-subtractive relationship, i.e.

$S(n+1) - S(n+2) = S(n+3) - S(n+4)$?

I found: $S(1) - S(2) = S(3) - S(4)$, $1 - 2 = 3 - 4$;

$S(2) - S(3) = S(4) - S(5)$, $2 - 3 = 4 - 5$;

$S(49) - S(50) = S(51) - S(52)$, $14 - 10 = 17 - 13$.

(c) How many sextuplets verify a Smarandache 3-3 additive relationship, i.e.

$S(n+1) + S(n+2) + S(n+3) = S(n+4) + S(n+5) + S(n+6)$?

I found: $S(5) + S(6) + S(7) = S(8) + S(9) + S(10)$, $5 + 3 + 7 = 4 + 6 + 5$.

I read that Charles Ashbacher has a computer program that calculates the Smarandache Function's values, therefore he may be able to add more solutions to mine.

More generally:

If f_p is a p-ary relation and g_q is a q-ary relation, both of them defined on

$\{ a_1, a_2, a_3, \ldots \}$, then

$a_{i1}, a_{i2}, \ldots, a_{ip}, a_{j1}, a_{j2}, \ldots, a_{jq}$

verify a *Smarandache* f_p - g_q - *relationship* if

$f_p(a_{i1}, a_{i2}, \ldots, a_{ip}) = g_q(a_{j1}, a_{j2}, \ldots, a_{jq})$.

If this relationship is verified by all terms of the sequence, then $\{a_n\}$, $n \geq 1$ is called a *Smarandache* f_p -g_q *-sequence*.

Study some Smarandache f_p -g_q - relationships for well-known sequences (perfect numbers, Ulam numbers, abundant numbers, Catalan numbers, Cullen numbers, etc.).

For example: a Smarandache 2-2-additive, or subtractive, or multiplicative relationship, etc.

If f_p is a p-ary relation on $\{ a_1, a_2, a_3, \ldots \}$ and

$f_p(a_{i1}, a_{i2}, \ldots, a_{ip}) = f_p\ a_{j1}, (a_{j2}, \ldots, a_{jp})$

for all a_{ik}, a_{jk}, where $k = 1, 2, \ldots, p$, and for all $p \geq 1$, then $\{a_n\}$, $n \geq 1$, is called a *Smarandache perfect f sequence.*

If not all p-plets $(a_{i1}, a_{i2}, \ldots, a_{ip})$ and $(a_{j1}, a_{j2}, \ldots, a_{jp})$ verify the f_p relation, or not for all $p \geq 1$, the relation f_p is verified, then $\{a_n\}$, $n \geq 1$ is called a *Smarandache partial perfect f-sequence.*

An example: a *Smarandache partial perfect additive sequence*:

1, 1, 0, 2, -1, 1, 1, 3, -2, 0, 0, 2, 1, 1, 3, 5, -4, -2, -1, 1, -1, 1, 1, 3, 0, 2, . . .

This sequence has the property that

$$\sum_{i=1}^{p} a_i = \sum_{j=p+1}^{2p} a_j,$$

for all $p \geq 1$.

It is constructed in the following way:

$$a_1 = a_2 = 1$$

$$a_{2p+1} = a_{p+1} - 1$$

$$a_{2p+2} = a_{p+1} + 1$$

for all $p \geq 1$.

(a) Can you, readers, find a general expression of a_n (as function of n)?

It is periodical, or convergent or bounded?

(b) Please design (invent) yourselves other Smarandache perfect (or partial perfect) sequences.

Think about a multiplicative sequence of this type.

2) Smarandache Subsequences

Let $\{a_n\}$, $n \geq 1$ be a sequence defined by a property (or a relationship involving its terms) P.

Now, we screen this sequence, selecting only its terms those digits hold the property (or relationship involving the digits) P.

The new sequence obtained is called:

(1) **Smarandache P-digital subsequences.**

For example:

(a) *Smarandache square-digital subsequence:*

0, 1, 4, 9, 49, 100, 144, 400, 441, . . .

i.e. from 0, 1, 4, 9, 16, 25, 36, ..., n^2, ... we choose only the terms whose digits are all perfect squares (therefore only 0, 1, 4, and 9).

Disregarding the square numbers of the form $\overline{N0 \ldots 0}$, where N is also a perfect square, how many other numbers belong to this sequence?
2k zeros

(b) *Smarandache cube-digital subsequence:*

0, 1, 8, 1000, 8000, . . .

i.e. from 0, 1, 8, 27, 64, 125, 216, . . . , n^3, . . . we choose only the terms whose digits are all perfect cubes (therefore only 0, 1 and 8).

Similar question, disregarding the cube numbers of the form $\overline{M0 \ldots 0}$ where M is a perfect cube.
3k zeros

(c) *Smarandache prime digital subsequence*:

2, 3, 5, 7, 23, 37, 53, 73, . . .

i.e. the prime numbers whose digits are all primes.

Conjecture: this sequence is infinite.

In the same general conditions of a given sequence, we screen it selecting only its terms whose groups of digits hold the property (or relationship involving the groups of digits) P.

[A group of digits may contain one or more digits, but not the whole term.]

The new sequence obtained is called:

(2) *Smarandache P-partial digital subsequence.*

Similar examples:

(a) *Smarandache square-partial-digital subsequence:*

49, 100, 144, 169, 361, 400, 441, . . .

i.e. the square members that is to be partitioned into groups of digits which are also perfect squares. (169 can be partitioned as $16 = 4^2$ and $9 = 3^2$, etc.)

Disregarding the square numbers of the form

$\overline{N0 \ldots 0}$, where N is also a perfect square,
2k zeros
how many other numbers belong to this sequence?

(b) *Smarandache cube-partial digital subsequence*:

1000, 8000, 10648, 27000, . . .

i.e. the cube numbers that can be partitioned into groups of digits which are also perfect cubes.

(10648 can be partitioned as $1 = 1^3$, $0 = 0^3$, $64 = 4^3$, and $8 = 2^3$).

Same question: disregarding the cube numbers of the form:

$\overline{M0 \ . \ . \ . \ 0}$ where M is also a perfect cube, how many other numbers belong
3k zeros
to this sequence?

(c) *Smarandache prime-partial digital subsequence*:

23, 37, 53, 73, 113, 137, 173, 193, 197, . . .

i.e. prime numbers, that can be partitioned into groups of digits which are also prime,

(113 can be partitioned as 11 and 3, both primes).

Conjecture: this sequence is infinite.

(d) *Smarandache Lucas-partial digital sunsequence*

123, . . .

i.e. the sum of the two first groups of digits is equal to the last group of digits, and the whole number belongs to Lucas numbers:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, . . .

(beginning at 2 and $L(n+2) = L(n+1) + L(n)$, $n \geq 1$) (123 is partitioned as 1, 2 and 3, then $3 = 2 + 1$).

Is 123 the only Lucas number that verifies a *Smarandache type partition*?

Study some Smarandache P - (partial) - digital subsequences associated to:

- Fibonacci numbers (we were not able to find any Fibonacci number verifying a Smarandache type partition, but we could not investigate large numbers; can you? Do you think none of them would belong to a Smarandache F - partial-digital subsequence?

- Smith numbers, Eulerian numbers, Bernoulli numbers, Mock theta numbers, Smarandache type sequences etc.

Remark: Some sequences may not be smarandachely partitioned (i.e. their associated Smarandache type subsequences are empty).

If a sequence $\{a_n\}$, $n \geq 1$ is defined by $a_n = f(n)$ (a function of n), then a *Smarandache f-digital subsequence* is obtained by screening the sequence and selecting only its terms that can be partitioned in two groups of digits g_1 and g_2 such that $g_2 = f(g_1)$.

(3) Study similar questions for this case, which is more complex.

An interesting law may be

$l(a_1, a_2, \ldots, a_n) = a_1 + a_2 - a_3 + a_4 - a_5 + \ldots$

Smarandache prime conjecture:

Any odd number can be expressed as the sum of two primes minus a third prime (not including the trivial solution p = p + q - q when the odd number is the prime itself).

For example:

1 = 3 + 5 - 7 = 5 + 7 - 11 = 7 + 11 - 17 = 11 + 13 - 24 = . . .
3 = 5 + 11 - 13 = 7 + 19 - 23 = 17 + 23 - 37 = . . .
5 = 3 + 13 - 11 = . . .
7 = 11 + 13 - 17 = . . .
9 = 5 + 7 - 3 = . . .
11 = 7 + 17 - 13 = . . .

(a) Is this conjecture equivalent to Goldbach's conjecture (any odd number ≥ 9 can be expressed as a sum of three primes - finally solved by Vinogradov in 1937 for any odd number greater than $_{3}3^{15}$)?

(b) The number of times each odd number can be expressed as a sum of two primes minus a third prime are called *Smarandache prime conjecture numbers*. None of them are known!

(c) Write a computer program to check this conjecture for as many positive odd numbers as possible.

(2) There are infinitely many numbers that cannot be expressed as the difference between a cube and a square (in absolute value).

They are called *Smarandache bad numbers*(!)

For example: 5, 6, 7, 10, 13, 14, . . . are probably such bad numbers (F. Smarandache has conjectured, see[1]), while

1, 2, 3, 4, 8, 9, 11, 12, 15, . . . are not, because

$1 = |2^3 - 3^2|$

$2 = |3^3 - 5^2|$

$3 = |1^3 - 2^2|$

$4 = |5^3 - 11^2|$

$8 = |1^3 - 3^2|$

$9 = |6^3 - 15^2|$

$11 = |3^3 - 4^2|$

$12 = |13^3 - 47^2|$

$15 = | 4^3 - 7^2 |$, etc.

(a) Write a computer program to get as many non Smarandache bad numbers (it's easier this way!) as possible,

i.e. find an ordered array of a's such that

$a = | x^3 - y^2 |$, for x and y integers ≥ 1.

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