

SMARANDACHE SEQUENCE OF TRIANGULAR NUMBERS

Shyam Sunder Gupta

Chief Engineer(C), S.E. Railway, Kolkata, India

Email: guptass@rediffmail.com

Web address: www.shyamsundergupta.com

Abstract:

In this article, we present the results of investigation of Smarandache Concatenate Sequence formed from the sequence of Triangular Numbers and report some primes and other results found from the sequence

Key words:

Triangular numbers, T-sequence, Smarandache T-sequence, Reversed Smarandache T-sequence, Prime.

1. Introduction:

Triangular numbers are formed by adding up the series $1+2+3+4+5+6+7+\dots$. The general formula for nth triangular number is given by $n(n+1)/2$. So, the sequence of triangular numbers starts [1]

1,3,6,10,15,21,28,36,45,55,66,78,91,105,120,136,153,171,190,.....

Let us denote the sequence of Triangular numbers as T-sequence. So, the sequence of Triangular numbers,

$T = \{ 1,3,6,10,15,21,28,36,45,55,66,78,91,105,120,136,153,171,190,\dots \}$.

2. Smarandache Sequence:

Let $S_1, S_2, S_3, \dots, S_n, \dots$ be an infinite integer sequence (termed as S-sequence), then the Smarandache sequence [2] or Smarandache Concatenated sequence [3] or Smarandache S-sequence is given by

$S_1, \overline{S_1S_2}, \overline{S_1S_2S_3}, \dots, \overline{S_1S_2S_3 \dots S_n}, \dots$

Also Smarandache Back Concatenated sequence or Reversed Smarandache S-sequence is

$S_1, \overline{S_2S_1}, \overline{S_3S_2S_1}, \dots, \overline{S_n \dots S_3S_2S_1}, \dots$

3. Smarandache T-Sequence:

Smarandache sequence of Triangular numbers or Smarandache T-sequence is the sequence formed from concatenation of numbers in T-sequence (Note that T-sequence is the sequence of Triangular numbers). So, Smarandache T-sequence is

1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836,

Let us denote the n^{th} term of the Smarandache T-sequence by $ST(n)$. So,

$$ST(1)=1$$

$$ST(2)=13$$

$$ST(3)=136$$

$$ST(4)=13610 \text{ and so on.}$$

3.1 Observations on Smarandache T-sequence:

We have investigated Smarandache T-sequence for the following two problems.

- (i) How many terms of Smarandache T-sequence are primes?
- (ii) How many terms of Smarandache T-sequence belongs to the initial T-sequence?

In search of answer to these problems, we find that

- (a) There are only 2 primes in the first 1000 terms of Smarandache T-sequence. These are $ST(2) = 13$ and $ST(6) = 1361011521$. It may be noted that $ST(1000)$ consists of 5354 digits.
- (b) Other than the trivial 1, there is only one Triangular number i.e. $ST(3)=136$, in first 1000 terms of Smarandache T-sequence and hence belongs to the initial T-sequence.

Open Problem:

- (i) Can you find more primes in Smarandache T-sequence and are there infinitely many such primes?
- (ii) Can you find more triangular numbers in Smarandache T-sequence and are there infinitely many such triangular numbers?

4.0 Reversed Smarandache T-Sequence:

It is defined as the sequence formed from the concatenation of triangular numbers (T-sequence) written backward i.e. in reverse order. So, Reversed Smarandache T-sequence is

1, 31, 631, 10631, 1510631, 211510631, 28211510631,

Let us denote the n^{th} term of the Reversed Smarandache T-sequence by $RST(n)$. So,

$$RST(1)=1$$

$$RST(2)=31$$

$$RSH(3)=631$$

$$RSH(4)=10631 \text{ and so on.}$$

4.1 Observations on Reversed Smarandache T-sequence:

(a) As against only 2 prime in Smarandache T-sequence, we found 6 primes in first 1000 terms of Reversed Smarandache T-sequence. These primes are:

$$\text{RSH}(2) = 31$$

$$\text{RSH}(3) = 631$$

$$\text{RSH}(4) = 10631$$

$$\text{RSH}(10) = 55453628211510631$$

$$\text{RSH}(12) = 786655453628211510631$$

$$\text{RSH}(14) = 10591786655453628211510631$$

(b) Other than the trivial 1, no Triangular number has been found in first 1000 terms of Reversed Smarandache T-sequence.

Open Problem:

- (i) Can you find more primes in Reversed Smarandache T-sequence and are there infinitely many such primes?
- (ii) Can you find triangular numbers in Reversed Smarandache T-sequence and are there infinitely many such triangular numbers?

REFERENCES

[1]. Sloane, N.J.A., Sequence A000217 (Triangular Numbers) in "The on line version of the Encyclopedia of Integer Sequences"

<http://www.research.att.com/~njas/sequences/>.

[2]. Weisstein, Eric W, "Triangular Number", "Consecutive Number Sequences" and "Smarandache Sequences", CRC Concise Encyclopedia of Mathematics, CRC Press, 1999.

[3]. Marimutha, H., "Smarandache Concatenate Type sequences", Bull. Pure Appl. Sci. 16E, 225-226, 1997.