SMARANDACHE SERIES CONVERGES

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The Smarandache Consecutive Series is defined by repeatedly concatenating the positive integers on the right side of the previous element.

1, 12, 123, 1234, . . . , 123456789, 12345678910, 1234567891011, . . .

The Smarandache Reverse Sequence is defined by repeatedly concatenating the positive integers on the left side of the previous element.

1, 21, 321, 4321, . . . , 987654321, 10987654321, 1110987654321, . . .

a) Consider the series formed by summing the inverses of the Smarandache Consecutive Series

1/1 + 1/12 + 1/123 + 1/1234 + . . .

It is a simple matter to prove that this series is convergent. Forming the series

1/1 + 1/10 + 1/100 + 1/1000 + . . .

where it is well-known that this series is convergent to the number 10/9. Furthermore, the elements of the two series matched in the following correspondence

1/1 <= 1/1, 1/12 <= 1/10, 1/123 <= 1/100, ...

Therefore,by the ratio test, the sum of the inverses of the Smarandache Consecutive Series is also convergent.

b) Consider the series formed by taking the ratios of the terms of the consecutive sequence over the reverse sequence.

1/1 + 12/21 + 123/321 + 1234/4321 + . . .

In this case, it is straightforward to show that the series is divergent.

Consider an arbitrary element of the sequence

$$e(n) = \frac{a_1a_2\ldots a_k}{a_k\ldots a_2a_1}$$

where the digit ak = 9. Clearly, e(n) > 1/10, as the numerator and denominator of this ratio have the same number of digits. Since there are an infinite number of such terms, the series contains an infinite number of terms all greater than 1/10. This forces divergence.