SMARANDACHE STAR (STIRLING) DERIVED SEQUENCES

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Let $b_1, b_2, b_3, \ldots$ be a sequence say S_b the base sequence. Then the **Smarandache star derived sequence** S_d using the following star triangle **{ref. [1]}** is defined

1				
1	1			
1	3	1		
1	7	6	1	
1	15	25	10	1

. . .

as follows

$d_1 = b_1$

$d_2 = b_1 + b_2$

$d_3 = b_1 + 3b_2 + b_3$

$d_4 = b_1 + 7b_2 + 6b_3 + b_4$

. . .

$$d_{n+1} = \sum_{k=0}^{n} a_{(m,r)} . b_{k+1}$$

where $a_{(m,r)}$ is given by

$$a_{(m,r)} = (1/r!) \sum_{t=0}^{r} (-1)^{r-t} . {}^{r}C_t . t^m \text{ , Ref. [1]}$$

e.g. (1) If the base sequence S_b is 1, 1, 1, . . . then the derived sequence S_d is 1, 2, 5, 15, 52, . . . , i.e. the sequence of Bell numbers. $T_n = B_n$

(2) $S_b \longrightarrow 1, 2, 3, 4, \ldots$ then

$S_d \longrightarrow 1, 3, 10, 37, \ldots$, we have $T_n = B_{n+1} - B_n$. **Ref [1]**

The Significance of the above transformation will be clear when we consider the inverse transformation. It is evident that the star triangle is nothing but the **Stirling Numbers of the Second kind (Ref. [2]).** Consider the inverse Transformation : Given the Smarandache Star Derived Sequence S_d , to retrieve the original base sequence S_b. We get b_k for $k = 1, 2, 3, 4$ etc. as follows ;

$b_1 = d_1$

$b_2 = -d_1 + d_2$

$b_3 = 2d_1 - 3d_2 + d_3$

$b_4 = -6d_1 + 11d_2 - 6d_3 + d_4$

$b_5 = 24d_1 - 50d_2 + 35d_3 - 10d_4 + d_5$

....

we notice that the triangle of coefficients is

1	
-1	1

2	-3	1		
-6	11	-6	1	
24	-50	35	-10	1

Which are nothing but the **Stirling numbers of the first kind.**

Some of the properties are

(1) The first column numbers are **$(-1)^{r-1}.(r-1)!$**, where r is the row number.

2. Sum of the numbers of each row is **zero.**
3. Sum of the absolute values of the terms in the r th row = r! .

More properties can be found in **Ref. [2].**

This provides us with a relationship between the Stirling numbers of the first kind and that of the second kind, which can be better expressed in the form of a matrix.

Let $[b_{1,k}]_{1xn}$ be the row matrix of the base sequence.

$[d_{1,k}]_{1xn}$ be the row matrix of the derived sequence.

$[S_{j,k}]_{nxn}$ be a square matrix of order n in which $s_{j,k}$ is the k^{th} number in the j^{th} row of the star triangle (array of the **Stirling numbers of the second kind** , Ref. [2]).

Then we have

$[T_{j,k}]_{nxn}$ be a square matrix of order n in which $t_{j,k}$ is the k^{th} number in the j^{th} row of the array of the **Stirling numbers of the first kind** , Ref. [2]). Then we have

$[b_{1,k}]_{1xn} * [S_{j,k}]'_{nxn} = [d_{1,k}]_{1xn}$

$[d_{1,k}]_{1xn} * [T_{j,k}]'_{nxn} = [b_{1,k}]_{1xn}$

Which suggests that $[T_{j,k}]'_{nxn}$ is the transpose of the inverse of the transpose of the Matrix $[S_{j,k}]'_{nxn}$.

The proof of the above proposition is inherent in **theorem 10.1 of ref. [3].**

Readers can try proofs by a combinatorial approach or otherwise.

REFERENCES:

[1] "Amarnath Murthy", 'Properties of the Smarandache Star Triangle' , SNJ, Vol. 11, No. 1-2-3, 2000.

[2] "V. Krishnamurthy" , 'COMBINATORICS Theory and applications' ,East West Press Private Limited, 1985.

[3] " Amarnath Murthy", 'Miscellaneous results and theorems on Smarandache Factor Partitions.', SNJ,Vol. 11,No. 1-2-3, 2000.