SMARANDACHE STRICTLY STAIR CASE SEQUENCE

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Given a number system with base 'b'. We define a sequence with the following postulates:

1. Numbers are listed in increasing order.
2. In a number the k^{th} digit is less than the $(k+1)^{th}$ digit.

Before we proceed with the general case, let us consider the case with b= 6. We get the following sequence.

1, 2, 3, 4, 5, 12, 13, 14, 15, 23, 24, 25, 34, 35, 45, 123, 124, 125, 134,135,145, 234, 235,245, 345, 1234, 1235, 1245, 1345, 2345, 12345.

For convenience we write the terms row wise with the r^{th} row containing numbers with r digits.

(1) 1, 2, 3, 4, 5, { ${}^5C_1 = 5$ numbers }
(2) 12, 13, 14, 15, 23, 24, 25, 34, 35, 45, { ${}^5C_2 = 10$ numbers }
(3) 123, 124, 125, 134,135,145, 234, 235,245, 345, { ${}^5C_3 = 10$ numbers }
(4) 1234, 1235, 1245, 1345, 2345, { ${}^5C_4 = 5$ numbers }
(5) 12345. { ${}^5C_5 = 1$ number }

Following properties can be noticed which are quite evident and can be proved easily.

**** We take (nothing) space as a number with zero number of digits.**

(1) There are ${}^{b-1}C_r$ (${}^5C_r$ in this case) numbers having exactly r digits.

(2) There are 2^{b-1} (2^5 =32, in this case) numbers in the finite sequence including the space which is considered as the lone number with zero digits.

3. The sum of the product of the digits of the numbers having exactly r digits is the absolute value of the r^{th} term in the b^{th} row of the array of the Stirling numbers of the First kind .

4. The sum of all the sums considered in (3) = b! - 1 (6! - 1 = 719 in this case) .

Open Problems:

1. **To derive an expression for the sum of all the r digit numbers and thus for the sum of the whole sequence.**
2. **We define the n^{th} number in the sequence to have index n. Given a number in the sequence to find it's index.**