Solutions To Some Sastry Problems On Smarandache Number Related Triangles

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In his recent paper[1], Sastry defines two triangles T(a,b,c) and T(a',b',c') to be Smarandache related if S(a) = S(a'), S(b) = S(b') and S(c) = S(c'). The function S is known as the Smarandache function and is defined in the following way.

For n any integer greater than zero, the value of the Smarandache function S(n) is the smallest integer m such that n divides m!.

He closes the paper by asking the following questions:

A) Are there two distinct dissimilar Pythagorean triangles that are Smarandache related? A triangle T(x,y,z) is Pythagorean if x*x + y*y = z*z.

B)Are there two distinct and dissimilar 60(120) degrees triangles that are Smarandache related? A 60(120) degrees triangle is one containing an angle of 60(120) degrees.

C) Given a triangle T(a,b,c), is it possible to give either an exact formula or an upper bound for the total number of triangles (without actually determining them), which are Smarandache related to T?

D) Consider other ways of relating two triangles in the Smarandache number sense. For example, are there two triplets of natural numbers (a,b,c) and (a',b',c') such that a + b + c = a' + b' + c' = 180 and S(a) = S(a'), S(b) = S(b') and S(c) = S(c')? If this were true, then the angles, in degrees, of the triangles would be Smarandache related.

In this paper, we will consider and answer questions (A), (B) and (D). Furthermore, we will also explore these questions using the Pseudo Smarandache function Z(n).

Given any integer n > 0, the value of the Pseudo Smarandache function is the smallest integer m such that n evenly divides

$$\sum_{k=1}^{m} k .$$

A) The following theorem is easy to prove.

Theorem: There are an infinite family of pairs of dissimilar Pythagorean triangles that are Smarandache related.

Proof:
Start with the two Pythagorean triangles

$T(3,4,5)$ and $T(5,12,13)$

Clearly, these two triangles are not similar. Now, let p be an odd prime greater than 13 and form the triples

$T(3p,4p,5p) \quad T(5p,12p,13p)$

Obviously, these triples are also Pythagorean. It is well-known that if $n = kp$, where $k < p$ and p is a prime, then $S(kp) = p$. Therefore,

$S(3p) = S(4p) = S(5p) = S(12p) = S(13p) = p$

and the triples form triangles that are not similar since the originals were not. Therefore, we have the desired infinite family of solutions.

Definition: Given two triangles $T(a,b,c)$ and $T(a',b',c')$, we say that they are Pseudo Smarandache related if $Z(a) = Z(a')$, $Z(b) = Z(b')$ and $Z(c) = Z(c')$.

A computer program was written to search for dissimilar pairs of Pythagorean triples $T(x,y,z)$ and $T(u,v,w)$ that are also Pseudo Smarandache related. Several were found and a few are given below.

$x = 49$, $y = 168$, $z = 175$
$Z(x) = 48$, $Z(y) = 48$, $Z(z) = 49$
$u = 147$, $v = 196$, $w = 245$
$Z(u) = 48$, $Z(v)=48$, $Z(w) = 49$

$x = 96$, $y = 128$, $z = 160$
$Z(x) = 63$, $Z(y) = 255$, $Z(z) = 64$
$u = 128$, $v = 504$, $w = 520$
$Z(u) = 255$, $Z(v) = 63$, $Z(w) = 64$

$x = 185$, $y = 444$, $z = 481$
$Z(x) = 74$, $Z(y) = 111$, $Z(z) = 221$
$u = 296$, $v = 555$, $w = 629$
$Z(u) = 111$, $Z(v) = 74$, $Z(w) = 221$

$x = 238$, $y = 816$, $z = 850$
$Z(x) = 84$, $Z(y) = 255$, $Z(z) = 424$
$u = 510$, $v = 680$, $w = 850$
$Z(u) = 84$, $Z(v) = 255$, $Z(w) = 424$

While these numbers do not readily display the pattern of an infinite family of solutions, there is no real reason to think that there is only a finite number of solutions.

Conjecture: There are an infinite number of pairs of Pythagorean triples T(x,y,z) and T(u,v,w) that are Pseudo Smarandache related.

C) A computer program was written to search for two dissimilar 60 degrees triangles T(a,b,c) and T(a1,b1,c1) that are Smarandache related and several solutions were found.

$a = 10$, $b = 14$, $c = 16$, $S(a) = 5$, $S(b) = 7$, $S(c) = 6$
$a1 = 30$, $b1 = 70$, $c1 = 80$, $S(a1) = 5$, $S(b1) = 7$, $S(c1) = 6$

$a = 10$, $b = 14$, $c = 16$, $S(a) = 5$, $S(b) = 7$, $S(c) = 6$
$a1 = 45$, $b1 = 105$, $c1 = 120$, $S(a1) = 6$, $S(b1) = 7$, $S(c1) = 5$

Note that the triangles T(30,70,80) and T(45, 105,120) are similar.

$a = 16$, $b = 19$, $c = 21$, $S(a) = 6$, $S(b) = 19$, $S(c) = 7$
$a1 = 80$, $b1 = 304$, $c1 = 336$, $S(a1) = 6$, $S(b1) = 19$, $S(c1) = 7$

$a = 20$, $b = 28$, $c = 32$, $S(a) = 5$, $S(b) = 7$, $S(c) = 8$
$a1 = 60$, $b1 = 140$, $c1 = 160$, $S(a1) = 5$, $S(b1) = 7$, $S(c1) = 8$

$a = 20$, $b = 28$, $c = 32$, $S(a) = 5$, $S(b) = 7$, $S(c) = 8$
$a1 = 120$, $b1 = 280$, $c1 = 320$, $S(a1) = 5$, $S(b1) = 7$, $S(c1) = 8$

Note again that the triangles T(60,140,160) and T(120, 280, 320) are similar. Given the number of solutions found in this limited search, the following conjecture seems safe.

Conjecture: There are an infinite number of dissimilar 60 degrees triangles that are Smarandache related.

Another computer program was written to search for dissimilar 60 degree triangles T(a,b,c) and T(a1,b1,c1) that are Pseudo Smarandache related. Only four pairs were found in a limited search and they are given below.

$a = 24$, $b = 56$, $c = 64$, $Z(a) = 15$, $Z(b) = 48$, $Z(c) = 127$
$a1 = 40$, $b1 = 56$, $c1 = 64$, $Z(a1) = 15$, $Z(b1) = 48$, $Z(c1) = 64$

a = 49, b = 91, c = 105, Z(a) = 48, Z(b) = 13, Z(c) = 14
a1 = 56, b1 = 91, c1 = 105, Z(a1) = 48, Z(b1) = 13, Z(c1) = 14

a = 42, b = 98, c = 112, Z(a) = 20, Z(b) = 48, Z(c) = 63
a1 = 70, b1 = 98, c1 = 112, Z(a1) = 20, Z(b1) = 48, Z(c1) = 63

a = 42, b = 98, c = 112, Z(a) = 20, Z(b) = 48, Z(c) = 63
a1 = 210, b1 = 294, c1 = 336, Z(a1) = 20, Z(b1) = 48, Z(c1) = 63

Question: Are there an infinite number of dissimilar 60 degrees triangles that are Pseudo Smarandache related?

Solutions to the corresponding problem for dissimilar 120 degrees triangles T(a,b,c) and T(a1,b1,c1) that are Smarandache related were also searched for using another computer program. Several were found, although they appear to be sparser than the corresponding 60 degrees triangles. The solutions that were found are as follows.

a = 32, b = 98, c = 78, S(a) = 8, S(b) = 14, S(c) = 13
a1 = 196, b1 = 364, c1 = 224, S(a1) = 14, S(b1) = 13, S(c1) = 8

a = 32, b = 98, c = 78, S(a) = 8, S(b) = 14, S(c) = 13
a1 = 392, b1 = 728, c1 = 448, S(a1) = 14, S(b1) = 13, S(c1) = 8

a = 51, b = 119, c = 17, S(a) = 17, S(b) = 17, S(c) = 17
a1 = 119, b1 = 221, c1 = 136, S(a1) = 17, S(b1) = 17, S(c1) = 17

a = 51, b = 119, c = 17, S(a) = 17, S(b) = 17, S(c) = 17
a1 = 238, b1 = 442, c1 = 272, S(a1) = 17, S(b1) = 17, S(c1) = 17

a = 51, b = 119, c = 17, S(a) = 17, S(b) = 17, S(c) = 17
a1 = 357, b1 = 663, c1 = 408, S(a1) = 17, S(b1) = 17, S(c1) = 17

a = 51, b = 119, c = 17, S(a) = 17, S(b) = 17, S(c) = 17
a1 = 272, b1 = 833, c1 = 663, S(a1) = 17, S(b1) = 17, S(c1) = 17

a = 51, b = 119, c = 17, S(a) = 17, S(b) = 17, S(c) = 17
a1 = 476, b1 = 884, c1 = 544, S(a1) = 17, S(b1) = 17, S(c1) = 17

Note the cases where the second triangles of pairs are similar.

Question: Is there an infinite family of dissimilar 120 degrees triangles that are Smarandache related?

Finding dissimilar 120 degrees triangles that are Pseudo Smarandache related proved to be more difficult. In a search for all $b \leq 337$, $a \leq 1000$, $c \leq 1000$, $a1 \leq 1000$,

$b1 \le 1000$ and $c1 \le 1000$, only one solution,

a = 168, b = 312, c = 192, Z(a) = 48, Z(b) = 143, Z(c) = 128
a1 = 192, b1 = 588, c1 = 468, Z(a) = 128, Z(b1) = 48, Z(c1) = 143

was found.

Question: Is there an infinite number of dissimilar 120 degrees triangles that are Pseudo Smarandache related?

D) A computer program was written to check for triplets of natural numbers (a,b,c) and (a',b',c') such that a + b + c = a' + b' + c' = 180 and S(a) = S(a'), S(b) = S(b') and S(c) = S(c') and many such pairs of triplets were found. While it is obvious that the number is finite, the following list is not exhaustive.

a = 1, b = 11, c = 168, S(a) = 0, S(b) = 11, S(c) = 7
a' = 1, b' = 14, c' = 165, S(a') = 0, S(b') = 7, S(c') = 11

a = 2, b = 7, c = 171, S(a) = 2, S(b) = 7, S(c) = 19
a' = 2, b' = 38, c' = 140, S(a') = 2, S(b') = 19, S(c') = 7

a = 3, b = 7, c = 170, S(a) = 3, S(b) = 7, S(c) = 17
a' = 6, b' = 21, c' = 153, S(a') = 3, S(b') = 7, S(c') = 17

The last being an example of a pair of triples where there is no number in common.

An exhaustive computer search revealed that all possible angle measures 1 through 178 can be an angle in such a pair of triangles except 83, 97, 107, 113, 121, 127, 137, 139, 149, 151, 163, 166, 167, 169, 172, 173, 174, 175, 176, 177, and 178.

The corresponding problem using the Pseudo Smarandache function is as follows.

Are there two triplets of natural numbers (a,b,c) and (a',b',c') such that a + b + c = a' + b' + c' = 180 and Z(a) = Z(a'), Z(b) = Z(b') and Z(c) = Z(c')?

Another computer program was written that used Z(n) rather than S(n) in the search for such triples. Many solutions exist and some are given below.

a = 2, b = 24, c = 154, Z(2) = 3, Z(24) = 15, Z(154) = 55
a' = 6, b' = 20, c' = 154, Z(6) = 3, Z(20) = 15, Z(154) = 55

a= 4, b = 8, c = 168, Z(4) = 7, Z(8) = 15, Z(168) = 48
a' = 4, b' = 56, c' = 120, Z(4) = 7, Z(56) = 48, Z(120) = 15

$a = 4$, $b = 16$, $c = 160$, $Z(4) = 7$, $Z(16) = 31$, $Z(160) = 64$
$a' = 14$, $b' = 62$, $c' = 104$, $Z(14) = 7$, $Z(62) = 31$, $Z(104) = 64$

The last solution shows us that there are solutions where there are no numbers common to the triples.

There are many solutions to this expression. An exhaustive computer search was performed for all possible values $1 \leq a \leq 178$ and the following numbers did not appear in any triple.

1, 15, 23, 35, 41, 45, 51, 59, 65, 67, 71, 73, 77, 79, 82, 83, 86, 87, 89, 90, 91, 97, 101, 102, 105, 107, 109, 113, 115, 116, 118, 121, 123, 125, 126, 127, 131, 134, 135, 137, 139, 141, 142, 143, 148, 149, 151, 152, 153, 157, 158, 159, 161, 163, 164, 166, 167, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178.

Reference

1. K. R. S. Sastry, 'Smarandache Number Related Triangles'. Appeared on the world wide web at http://www.gallup.unm.edu/~smarandache.