SOME CONJECTURES ON PRIMES (II)

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Abstract. For any positive integer n, let p_n be the nth prime. In this paper we give a counter-example for the inequality
$\exp\left(\sqrt{(n+1)/p_{n+1}}\right)/\exp\left(\sqrt{p_n/n}\right) < \exp\left(\sqrt{3/5}\right)/\exp\left(\sqrt{3/2}\right)$

Key words: prime, inequality

For any positive integer n, let p_n be the nth prime. Recently, Russo [3] proposed the following conjecture:

Conjecture For any positive integer n,

$$(1) \qquad |p_n \cdot (n+1) - n \cdot p_{n+1}| < \frac{1}{2}(n+1)^{9/50}.$$

In [3], Russo verified the equality (1) holds for $p_n \leqslant 10^7$. However, we shall show that (1) is false for some n.

Let p_n and p_{n+1} be twin primes. Then we have

$$(2) \qquad p_{n+1} = p_n + 2.$$

If (1) holds, then from (2) we get

$$(3) \qquad |p_n - 2n| < \frac{1}{2}(n+1)^{9/50}.$$

It is a well known fact that

$$(4) \qquad p_n > n \log n$$

for any positive integer n (see [2]). Therefore, by (3) and (4), we obtain

$$(5) \qquad n(\log n - 2) < \frac{1}{2}(n+1)^{9/50}, n > 6.$$

By [1], p_n=297.2^{546}-1 and p_n+1=297.2^{546}+1 are twin primes. Then we have $n > 10^{10}$. Therefore, (5) is impossible. Thus, the inequality (1) is false for some n.

References

[1] R. Baillie, New primes of the form $k \cdot 2^n+1$, Math. Comp. 33(1979), 1333-1336.

[2] B. Rosser, The nth prime is greater than $n\log n$, Proc. London Math. Soc. (2) 45(1938), 21-44.

[3] F. Russo, Ten conjectures on prime numbers, Smarandache Notions J. 12(2001), 295-296.

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