SOME MORE CONJECTURES ON PRIMES AND DIVISORS

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There are an innumerable numbers of conjctures and unsolved problems in number theory predominantly on primes which have been giving sleepless nights to the mathematicians allover the world for centuries. Here are a few more to trouble them.

(1) Every even number can be expressed as the difference of two primes.

(2) Every even number can be expressed as the diference of two consecutive primes.

i.e. for every **m** there exists an **n** such that $2m = p_{n+1} - p_n$., where p_n is the n^{th} prime.

(3) Every number can be expressed as N / d(N) , where d(N) is the number of divisors of N.

If d(N) divides N we define N / d(N) = I as the **index of beauty** for N.

The conjecture can be stated in other words as follows. For every natural number M there exists a number N such that M is the index of beauty for N. i.e. M = N/d(N) .

The conjecture is true for primes can be proved as follows:

We have 2 = 12 / d(12) = 12/ 6 , 2 is the index of beauty for 12.

3 = 9 /d(9) = 9 / 3 , 3 is the index of beauty for 9.

For a prime p >3 we have N = 12p , d(N) = 12 and N /d(N) = p .

(N= 8p can also be taken) .

The conjectue is true for a large number of canonical forms can be established and further explored.

The proof for the geneal case or giving a counter example is still a challengeing unsolved problem..

(4) If p is a prime there exist infinitely many primes of the form

A. $2^n p + 1$. (B) $2.a^n p + 1$.

(5) It is a well known fact that one can have arbitrarily large numbers of consecutive composite numbers.

i.e. (r+1)! +2 , (r+1)! +3 , (r+1)! +4 , . . .(r+1)! + r-1 , (r+1)! + r give **r** cosecutive compsite number where r is chosen arbitrarily.

But these are not necessarrily the smallest set of such numbers. Let us consider the smallest set of r consecutive composite numbers as follows

r	Smallest set of compsite numbers	r / first compsite number
1	1	1/1
2	8 ,9	2/8
3	14, 15, 16	3/14
4	24, 25, 26, 27	4/24
5	24, 25, 26, 27, 28	5/24
6	90, 91, 92, 93, 94, 95	6/90
7	90, 91, 92, 93, 94, 95, 96	7/90
8	114 ,115,. . up to. . 121	8/114

Similarly for 9 , 10 , 11, 12, 13 the fisrt of the composite nmbers is 114.

We conjecture that the sum of the ratios in the third column is finite and > e.

(6) Given a number N . Carryout the following step of operation to get a number N_1

$N - p_{r1} = N_1$, where $p_{r1} < N < p_{r1+1}$, p_{r1} is the r_1^{th} prime .

Repeat the above step to get N_2

$N_1 - p_{r2} = N_2$, $p_{r2} < N_1 < p_{r2+1}$.

Go on repeating these steps till one gets $N_k = 0$ or 1.

The conjecture is (a) however large N be , k < $\log_2 \log_2 N$

(b) There exists a constant C such that k < C.

Open Problem: In case (b) is true , find the value of C.