SOME MORE IDEAS ON SMARANDACHE FACTOR PARTITIONS

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ABSTRACT: In [1] we define SMARANDACHE FACTOR PARTITION FUNCTION (**SFP**) , as follows:

Let $\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r$ be a set of r natural numbers and $p_1 , p_2, p_3 , \ldots p_r$ be arbitrarily chosen distinct primes then $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ called the Smarandache Factor Partition of $(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r)$ is defined as the number of ways in which the number

$$N = p_1^{\alpha 1} \; p_2^{\alpha 2} \; p_3^{\alpha 3} \ldots p_r^{\alpha r}$$ could be expressed as the product of its' divisors. For simplicity , we denote $F(\alpha_1 , \alpha_2 , \alpha_3 , \ldots \alpha_r) = F'(N)$,where

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r} \ldots p_n^{\alpha_n}$$

and p_r is the r^{th} prime. $p_1 =2$, $p_2 = 3$ etc.

In this note another result pertaining to SFPs has been derived.

DISCUSSION:

Let

$$N = p_1^{\alpha_1} \; p_2^{\alpha_2} \; p_3^{\alpha_3} \ldots p_r^{\alpha_r}$$

(1) L(N) = length of that factor partition of N which contains the maximum number of terms. In this case we have

$$L(N) = \sum_{i=1}^{r} \alpha_i$$

(2)

$A_{L(N)}$ = A set of L(N) distinct primes.

(3) B(N) = { p: p | N , p is a prime. }

$B(N) = \{ p_1 ; p_2 , \ldots p_r \}$

(4) $\Psi[N, A_{L(N)}] = \{ x \mid d(x) = N \text{ and } B(x) \subseteq A_{L(N)} \}$, where d(x) is the number of divisors of x.

To derive an expression for the order of the set $\Psi[N, A_{L(N)}]$ defined above.

There are F'(N) factor partitions of N. Let F_1 be one of them.

$F_1 \dashrightarrow N = s_1 \times s_2 \times s_3 \times \ldots \times s_t.$

if

$$\theta = p_1^{s_1-1} \; p_2^{s_2-1} \; p_3^{s_3-1} \ldots p_t^{s_t-1} \; p_{t+1}^{0} \; p_{t+2}^{0} \ldots p_{L(N)}^{0}$$

where $p_t \in A_{L(N)}$, then $\theta \in \Psi[N, A_{L(N)}]$ for

$d(\theta) = s_1 \times s_2 \times s_3 \times \ldots \times s_t \times 1 \times 1 \times 1 \ldots = N$

Thus each factor partition of N generates a few elements of Ψ .

Let $E(F_1)$ denote the number of elements generated by F_1

$F_1 \dashrightarrow N = s_1 \times s_2 \times s_3 \times \ldots \times s_t.$

multiplying the right member with unity as many times as required to make the number of terms in the product equal to L(N) .

$$N = \prod_{k=1}^{L(N)} s_k$$

where $s_{t+1} = s_{t+2} = s_{t+3} = \ldots = s_{L(N)} = 1$

Let x_1 s's are equal

x_2 s's are equal

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x_m s's are equal

such that $x_1 + x_2 + x_3 + \ldots + x_m = L(N)$. Where any x_i can be unity also.

Then we get

$$E(F_1) = \{L(N)\}! \,/\, \{(x_1)!(x_2)!(x_3)! \ldots (x_m)!\}$$

summing over all the factor partitions we get

$$\mathbf{O(\Psi[\,N, A_{L(N)}])} = \sum_{k=1}^{F'(N)} \mathbf{E(F_k)} \quad \text{-----------}\mathbf{(7.1)}$$

Example:

$N = 12 = 2^2.3$, $L(N) = 3$, $F'(N) = 4$

Let $A_{L(N)} = \{ 2,3,5\}$

$F_1 \text{-----}\rightarrow N = 12 = 12 \times 1 \times 1$, $x_1 = 2$, $x_2 = 1$

$E(F_1) = 3! \,/\, \{(2!)(1!)\} = 3$

$F_2 \text{-----}\rightarrow N = 12 = 6 \times 2 \times 1$, $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

$E(F_2) = 3! \,/\, \{(1!)\,(1!)(1!)\} = 6$

$F_3 \text{-----}\rightarrow N = 12 = 4 \times 3 \times 1$, $x_1 = 1$, $x_2 = 1$, $x_3 = 1$

$E(F_3) = 3! \,/\, \{(1!)\,(1!)(1!)\} = 6$

$F_4 \text{-----}\rightarrow N = 12 = 3 \times 2 \times 2$, $x_1 = 1$, $x_2 = 2$

$E(F_4) = 3! \,/\, \{(2!)(1!)\} = 3$

$$O(\Psi[N, A_{L(N)}]) = \sum_{k=1}^{F'(N)} E(F_k) = 3 + 6 + 6 + 3 = 18$$

To verify we have

$\Psi[N, A_{L(N)}] = \{ 2^{11}, 3^{11}, 5^{11}, 2^5 X 3, 2^5 X 3, 3^5 X 2, 3^5 X 5, 5^5 X 2,$
$5^5 X 3, 2^3 X 3^2, 2^3 X 5^2, 3^3 X 2^2, 3^3 X 5^2, 5^3 X 2^2, 5^3 X 3^2, 2^2 X 3 X 5,$
$3^2 X 2 X 5, 5^2 X 2 X 3, \}$

REFERENCES:

[1] "Amarnath Murthy" , 'Generalization Of Partition Function, Introducing 'Smarandache Factor Partition', SNJ, Vol. 11, No. 1-2-3, 2000.

[2] " The Florentine Smarandache " Special Collection, Archives of American Mathematics, Centre for American History, University of Texas at Austin, USA.