SOME PROBLEMS CONCERNING THE SMARANDACHE SQUARE COMPLEMENTARY FUNCTION (I)

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Abstract: For any positive integer n, let $SSC(n)$ denote the Smarandache square complementary function of n. In this paper we prove that the difference $|SSC(n+1)-SSC(n)|$ is unbounded.

Key words: Smarandache square complementary function; difference; Pell equation

For any positive integer n, let $SSC(n)$ denote the least positive integer m such that mn is a perfect square. Then $SSC(n)$ is called the Smarandache square complementary function (see [1]). In [3], Russo asked if the difference

$$|SSC(n+1)-SSC(n)| \tag{1}$$

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is bounded or unbounded? In this paper we solve this problem as follows.

Theorem. The difference is unbounded.

Proof. Let d be a positive integer with square free. By [2, Theorem 10.9.1], there exist two positive integers x and y such that

$$x^2-dy^2=1. \tag{2}$$

Let $n=dy^2$. Then from (2) we get $n+1=x^2$. By the define of the Smarandache square complementary function, we have

$$SSC(n)=d,\ SSC(n+1)=1. \tag{3}$$

Therefore, by (3), we get

$$|SSC(n+1)-SSC(n)|=d-1. \tag{4}$$

Since there exist infinitely many positive integers d with square free, we see from (4) that the difference (1) is unbounded. Thus, the theorem is proved.

References

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[3] F.Russo, An introduction to the Smarandache square complementary function, Smarandache Notions J. 13 (2002), 160-173.