SOME PROBLEMS CONCERNING THE SMARANDACHE SQUARE COMPLEMENTARY FUNCTION (II)

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Abstract: In this paper we solve three diophantine equations concerning the Smarandache square complementary function.

Key words: Smarandache square complementary function; diophantine equations

For any positive integer n, let $SSC(n)$ denote the Smarandache square complementary function of n (see [1]). In [2], Russo proposed three problems concerning the equations

$$SSC(n) = SSC(n+1) \cdot SSC(n+2), \tag{1}$$

$$SSC(n) \cdot SSC(n+1) = SSC(n+2), \tag{2}$$

and

$$SSC(n) \cdot SSC(n+1) = SSC(n+2)SSC(n+3), \tag{3}$$

In this paper we completely solve these problems as follows.

Supported by the National Natural Science Foundation of China (No.10271104), the Guangdong Provincial Natural Science Foundation (No.011781) and the Natural Science Foundation of the Education Department of Guangdong Province (No.0161).

Theorem. The equations (1), (2) and (3) have no positive integer solutions n.

Proof. Let n be a positive integer solution of (1). Then from (1) we get

$$SSC(n) \equiv 0 \pmod{SSC(n+1)}. \tag{4}$$

By [2, Theorem 6], we have

$$n \equiv 0 \pmod{SSC(n)}, \quad n+1 \equiv 0 \pmod{SSC(n+1)}. \tag{5}$$

Since gcd $(n, n+1)=1$, we get from (5) that

$$\gcd(SSC(n), SSC(n+1))=1. \tag{6}$$

Hence, by (4) and (6), we obtain $SSC(n+1)=1$. It implies that $n+1=m^2$, where m is a positive integer.

If m is even, then n is odd and gcd $(n, n+2)=1$. It follows that

$$\gcd(SSC(n), SSC(n+2))=1. \tag{7}$$

Since $SSC(n+1)=1$, we get from (1) that

$$SSC(n)=SSC(n+2). \tag{8}$$

The combination of (7) and (8) that $SSC(n)=SSC(n+2)=1$. It implies that $n=l^2$, where l is a positive integer. But, since $n+1=m^2$, it is impossible.

If n is odd, then gcd$(n, n+2)=2$. Since $SSC(n+1)=1$, then (8) holds and $SSC(n)=SSC(n+2)=2$. It implies that

$$n=2x^2, \quad n+2=2y^2, \tag{9}$$

where x,y are positive integers. Buy, by (9), we obtain $y^2=x^2+1$, a contradiction. Thus, the equation (1) has no positive integer solution n.

By the same argument, we can prove that (2) and (3) have no positive integer solutions n. The theorem is proved.

References

[1] C.Dumitrescu and V. Seleacu, Some notions and questions in number theory, Xiquan Pub. House, Phoenix-Chicago, 1994.

[2] F.Russo, An introduction to the Smarandache square complementary function, Smarandache Notions J. 13(2002), 160-173.