SOME PROBLEMS CONCERNING THE SMARANDACHE SQUARE COMPLEMENTARY FUNCTION (III)

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Abstract: In this paper we discuss a diophantine equations concerning the Smarandache square complementary function.

Key words: Smarandache square complementary function; diophantine equations

For any positive integer n, let $SSC(n)$ denote the Smarandache square complementary function of n (see [1]). In [2], Russo asked that if the equation

$$SSC(mn) = m^k SSC(n), \tag{1}$$

has positive integer solutions (m, n, k). In this paper we prove the following result.

Theorem. The positive integer solutions (m, n, k) of (1) satisfy $k=1$. Moreover, (1) has infinitely many positive integer solutions $(m, n, k)=(a, b, 1)$ with $k=1$, where a,b are coprime positive integer with square free.

Supported by the National Natural Science Foundation of China (No.10271104), the Guangdong Provincial Natural Science Foundation (No.011781) and the Natural Science Foundation of the Education Department of Guangdong Province (No.0161).

Proof. Let (m, n, k) be a positive integer solution of (1). Further, let $d=\gcd(m, n)$. Then we have

$$m=da, \quad n=db, \tag{2}$$

where a,b are coprime positive integers. Substitute (2) into (1), we get

$$\begin{aligned} SSC(mn) &= SSC(d^2ab) = SSC(ab) = SSC(a)SSC(b) \\ &= (da)^k SSC(db), \end{aligned} \tag{3}$$

since $\gcd(a, b)=1$. By (3), we have

$$SSC(a)SSC(b) \equiv 0 \pmod{a^k}. \tag{4}$$

It is a well known fact that

$$a \equiv 0 \pmod{SSC(a)},\ b \equiv 0 \pmod{SSC(b)}. \tag{5}$$

Since $\gcd(a, b)=1$, we see from (4) and (5) that

$$SSC(a) \equiv 0 \pmod{a^k}. \tag{6}$$

Further, since $SSC(a) \le a$, we find from (6) that $k=1$. It implies that the solutions (m, n, k) of (1) satisfy $k=1$.

On the other hand, if a and b are coprime positive integers with square free, then we have

$$SSC(ab)=SSC(a)SSC(b)=aSSC(b). \tag{7}$$

It implies that $(m, n, k)=(a, b, 1)$ is a positive integer solution of (1). Thus, the theorem is proved.

References

[1] C.Dumitrescu and V. Seleacu, Some notions and questions in number theory, Xiquan Pub. House, Phoenix-Chicago, 1994.

[2] F.Russo, An introduction to the Smarandache square complementary function, Smarandache Notions J. 13(2002), 160-173.