SOME PROBLEMS CONCERNING THE SMARANDACHE SQUARE COMPLEMENTARY FUNCTION (V)

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Abstract: In this paper we discuss the convergence for two series concerning the Smarandache square complementary function.

Key words: Smarandache square complementary function; series; convergence

For any positive integer n, let $SSC(n)$ denote the Smarandache square complementary function of n (see [1]). Let

$$S_1 = \sum_{n=1}^{\infty} \frac{1}{SSC(n)^a}, \tag{1}$$

$$S_2 = \sum_{n=1}^{\infty} (-1)^n \frac{1}{SSC(n)}, \tag{2}$$

where a is a positive number. In [2], Russo proposed two problems concerning the convergence of the series (1) and (2). In this paper we prove the following two results.

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Theorem 1. If $a \le 1$, then S_1 is divergence.

Theorem 2. The series S_2 is divergence.

Proof of Theorem 1. Let $\zeta(z)$ denote the Riemann ζ-function. Then we have

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}, \tag{3}$$

if z is a positive number. It is a well known fact that $SSC(n) \le n$ for any n. Hence, by (1) and (3), we get

$$S_1 \ge \zeta(a). \tag{4}$$

Notice that $\zeta(a)$ is divergence if $a \le 1$. Thus, we see from (4) that S_1 is divergence if $a \le 1$. The theorem is proved.

Proof of Theorem 2. Let

$$S = \sum_{m=0}^{\infty} \frac{1}{SSC(2m+1)}. \tag{5}$$

We see from (2) that

$$S_2 = \sum_{n=1}^{\infty} (-1)^n \frac{1}{SSC(n)} = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} (-1)^{2^k(2m+1)} \frac{1}{SSc\left(2^k(2m+1)\right)}. \tag{6}$$

Since

$$SSC\left(2^k(2m+1)\right) = \begin{cases} SSC(2m+1), & \text{if } k \text{ is even}, \\ 2SSC(2m+1), & \text{if } k \text{ is odd}, \end{cases} \tag{7}$$

we get from (5), (6) and (7) that

$$S_2 = -S + \frac{1}{2}S + S + \frac{1}{2}S + S + \cdots. \tag{8}$$

It implies that S_2 is divergence. The theorem is proved.

References

[1] C.Dumitrescu and V. Seleacu, Some notions and questions in number theory, Xiquan Pub. House, Phoenix-Chicago, 1994.

[2] F.Russo, An introduction to the Smarandache square complementary function, Smarandache Notions J. 13(2002), 160-173.