

SOME RESULTS CONCERNING THE SMARANDACHE DECONSTRUCTIVE SEQUENCE

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ABSTRACT

In this note some new primes which were found in the Smarandache Deconstructive Sequence (SDS(n)) are reported, unusual sequences involving SDS(n) are given, along with a list of factorizations for SDS(n). All computations were done with PARI/GP [3], except where noted.

I INTRODUCTION

The Smarandache Deconstructive Sequence, SDS(n) (A007923) [4] is:

1, 23, 456, 7891, 23456, 789123, 4567891, 23456789, 123456789, ...

in which the lengths of the terms increase by 1, and the digits sequentially repeat 1-9. Smarandache first defined this sequence in [5].

II PRIMES IN SDS(n)

In [1] Ashbacher listed eight primes that arise in the Smarandache Deconstructive Sequence:

23, 4567891, 23456789, 1234567891, 23456789123456789,  
23456789123456789123, 4567891234567891234567891,  
1234567891234567891234567891

The author has found five more.

The values of n for which SDS(n) is prime are:

2, 7, 8, 10, 17, 20, 25, 28, 31, 38, 61, 62, 355,

with no more terms being found for  $n \leq 500$ .

For example,  $SDS(28) = 1234567891234567891234567891$  which is the last prime in Ashbacher's list.

The largest prime the author found,  $SDS(355) =$

789123456789123456789123456789123456789123456789123456\

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 789123456789123456789123456789123456789123456\  
 7891234567891234567891234567891234567891

has been proven prime with Primo [2].

### III SOME UNUSUAL SEQUENCES INVOLVING SDS(n)

If we sum the squares of the individual digits of SDS(n) we get the following sequence:

(a) 1, 13, 77, 195, 90, 208, 272, 284, 285, 286, 298, 362, 480, 375, 493, 557,

For example,  $1^2 = 1$ ;  $2^2 + 3^2 = 13$ ;  $4^2 + 5^2 + 6^2 = 77$ ;  
 $7^2 + 8^2 + 9^2 + 1^2 = 195$ , etc.

Are there any squares in sequence (a) above?

Yes. For the following values of n the sum of the squares of the digits of SDS(n) is a square:

1, 100, 280, 346, 568, 721, 1021, 1153, 1657, 2548, 2565,  
 2584, 3673, 4537, 4801, 5545, 6004, 6826, 7156,

Is this sequence infinite?

Another question one might ask concerning sequence (a) is, will any primes occur?

For the following values of n the sum of the squares of the digits of SDS(n) is prime:

2, 16, 17, 19, 21, 33, 38, 39, 52, 53, 56, 57, 69, 70, 73, 74,  
 75, 88, 91, 93, 105, 106, 110, 125, 128, 141, 142, 145, 147,  
 177, 181, 196, 197, 199, 213, 214, 217, 219, 231, 235, 237,  
 254, 268, 272, 273, 285, 290, 303, 304, 305, 309, 322, 323, ...

We conjecture that this sequence is infinite.

If we sum the individual digits of SDS(n) after raising the digit to its own power we get the following sequence:

(b) 1, 31, 50037, 405021249, 50068, 405021280, 405071286, 405071316, ...

For example,  $1^1 = 1$ ;  $2^2 + 3^3 = 31$ ;  $4^4 + 5^5 + 6^6 = 50037$ , etc.

After searching for primes in sequence (b), these values of n such that the sum of the digits of SDS(n) when raised to their own power is prime were found:

2, 21, 32, 33, 69, 92, 93, 94, 107, 123, 140, 163, 164, 248, 269,

272, 291, 307, 326, 345, 364, 377, 392, 393, 433, 434, 448, 453,  
454, 485, 487, 502, 519, 538, 573, 580, 626, 627, 685, 718, 755,  
757, 809, 865, 866, 878, 917, 955, 973, 986, 988, 1024, 1028, 1048,

We conjecture that this sequence is infinite.

#### IV FACTORS OF SDS(n)

In closing, we provide a list of factors for the first fifty values of SDS(n).

SDS1:  
0  
SDS2:  
23  
SDS3:  
2<sup>3</sup>.3.19  
SDS4:  
13.607  
SDS5:  
2<sup>5</sup>.733  
SDS6:  
3.17.15473  
SDS7:  
4567891  
SDS8:  
23456789  
SDS9:  
3<sup>2</sup>.3607.3803  
SDS10:  
1234567891  
SDS11:  
59.397572697  
SDS12:  
2<sup>7</sup>.3.23.467.110749  
SDS13:  
37.353.604183031  
SDS14:  
2<sup>7</sup>.13.23.47.13040359  
SDS15:  
3.19.13844271171739  
SDS16:  
739.1231.4621.1086619  
SDS17:  
23456789123456789  
SDS18:  
3<sup>2</sup>.7.11.13.19.3607.3803.52579  
SDS19:  
31.241.1019.162166841159  
SDS20:  
23456789123456789123  
SDS21:  
2<sup>7</sup>.3.19.83.67247.11217082711  
SDS22:  
13.1171.5009.103488876927413  
SDS23:

2^7.37139.4934332239074993  
SDS24:  
3.29.53.19447.5949239.1479230321  
SDS25:  
4567891234567891234567891  
SDS26:  
31.120817.6262948234815488507  
SDS27:  
3^3.757.3607.3803.440334654777631  
SDS28:  
1234567891234567891234567891  
SDS29:  
20393.16338731.70399426574704481  
SDS30:  
2^7.3.43.27664069976791976953536163  
SDS31:  
7891234567891234567891234567891  
SDS32:  
2^7.13.67.439.166657.2875758147251799619  
SDS33:  
3.19.43.191.1685653375348716426865246703  
SDS34:  
3671.14074661.88408378782858625690561  
SDS35:  
11083.590819.9973889.119776913.2998604101  
SDS36:  
3^2.7.11.13.19.101.3607.3803.9901.52579.999999000001  
SDS37:  
5077076293.243165124963105984043672887  
SDS38:  
23456789123456789123456789123456789123  
SDS39:  
2^7.3.19.62608158368529211000108158368529211  
SDS40:  
13.1794115880987.3016274343701.112170916993561  
SDS41:  
2^7.3547.19141.1822695439.1480875933915409449259  
SDS42:  
3.36677.77890601.6953106199727.13242377845224779  
SDS43:  
17.268699484386346543209875954974581837327523  
SDS44:  
17.911.98981.9659394263.240869841259.6576837459611  
SDS45:  
3^2.31.41.271.3607.3803.238681.2906161.4185502830133110721  
SDS46:  
47.15667.62788723633.26702358442667031058467275423  
SDS47:  
857.27370815779996253352925074823170115663310139  
SDS48:  
2^7.3.157089311.7572475819198513188475662796700757119  
SDS49:  
17.593.138599.31074683.1398187430503.129989759693807375161  
SDS50:  
2^7.13^2.461.66173.3920187843941.9067410177727179700576871

## REFERENCES

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