SOME SMARANDACHE-TYPE SEQUENCES AND PROBLEMS CONCERNING ABUNDANT AND DEFICIENT NUMBERS

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Abstract: We define some new sequences involving Smarandache operations on the sets of deficient and abundant numbers. We give conjectures and ask questions about these sequences somewhat similar to certain problems posed in Smarandache's book *Only Problems, Not Solutions!* [7].

1. INTRODUCTION

A number n is called abundant if $\sigma(n) > 2n$ (A005101), perfect if $\sigma(n) = 2n$ (A000396), and deficient if $\sigma(n) < 2n$ (A005100), where $\sigma(n)$ denotes the sum of all positive divisors of n (A000203) [6]. Concerning perfect numbers, it is not known when they were first studied, however, the first mathematical result about them occurs in Euclid's *Elements* written around 300 BC. More relevant to this paper is the text *Introductio Arithmetica*, written by Nicomachus around 100 AD, in which Nicomachus first classified all numbers based on the concept of perfect numbers, thus giving us the definitions (listed above) of abundant and deficient numbers, with which this paper deals [4].

Concerning abundant numbers, two of the more interesting facts about them is that in 1964 T. R. Parkin and L. J. Lander showed that all numbers greater than 20161 can be expressed as the sum of two abundant numbers [2]; and around 1000 AD, Abu Mansur ibn Tahir Al-Baghadadi found the first smallest odd abundant number: 945 [5].

Perfect numbers have attracted more interest through the years than abundant and deficient numbers, no doubt due to the fact that they are intimately connected with Mersenne primes. But despite all of the extensive study of perfect numbers, there are still crucial unsolved problems. For example: Are there infinitely many perfect numbers? Does an odd perfect number exist? No one knows. But no matter how much more attractive the perfect numbers may seem when compared with the abundant and deficient numbers, in this paper we leave the perfect ones alone and devote our energy only to the abundants and deficients.

What we offer in this paper are some Smarandache-type sequences and problems with the questions asked being very much in the spirit of Florentin Smarandache's wonderful book *Only Problems, Not Solutions!* [7]. Also it should be mentioned that in constructing and exploring the problems below we made extensive use of the software package PARI/GP [3]; and that all of the conjectures made were based on a small amount of analysis and a lot of empirical evidence via a personal computer. And now we close this introduction with a quote from Sir Winston Churchill (1874-1965):

> "I had a feeling once about Mathematics - that I saw it all. Depth beyond depth was revealed to me - the Byss and Abyss. I saw - as one might see the transit of Venus or even the Lord Mayor's Show - a quantity passing through infinity and changing its sign from plus to minus. I saw exactly why it happened and why the tergiversation was inevitable, but it was after dinner and I let it go" [1].

2. SEQUENCES AND PROBLEMS

(1) Smarandache Consecutive Abundant sequence (SCA); concatenate the first n abundant numbers.

12, 1218, 121820, 12182024, 1218202430, 121820243036, 12182024303640, 1218202430364042, 121820243036404248, 12182024303640424854,...

Will there always be at least one prime factor of any SCA number that has never before appeared as a prime factor in any earlier SCA number? That is, if SCA = $p_1^{a1} p_2^{a2} \ldots p_n^{an}$, is their always a p_i in any SCA number distinct from all previous SCA numbers? We conjecture: yes.

(2) Smarandache Consecutive Deficient sequence (SCD); concatenate the first n deficient numbers.

1, 12, 123, 1234, 12345, 123457, 1234578, 12345789, 1234578910, 123457891011, 12345789101113, 1234578910111314, 123457891011131415,...

How many primes are among these numbers? Will there always be at least one prime factor of any SCD number > 1 that has never before appeared as a prime factor in any earlier SCD number? We conjecture: yes.

(3) Smarandache Abundant-Deficient consecutive sequence; a_d where a is the nth abundant number and d is the nth deficient number, with "_" representing concatenation.

121, 182, 203, 244, 305, 367, 408, 429, 4810, 5411, 5613, 6014, 6615, 7016, 7217, 7819, 8021, 8422, 8823, 9025, 9626, 10027, 10229, 10431, 10832, 11233, 11434, 12035, 12637, 13238,...

How many primes are there among these numbers? How many squares?

(4) Smarandache Odd Abundant-Deficient consecutive sequence; oa_od where oa is the nth odd abundant number and od is the nth odd deficient number, with "_" representing concatenation.

9451,15753,22055,28357,34659,409511,472513,535515,577517, 598519,643521,661523,682525,724527,742529,787531,808533,...

We conjecture that there are an infinite amount of primes among these numbers. How many of these numbers are triangular?

(5) Deficient numbers such that the sum of their individual digits after being raised to their own power, become abundant numbers.*

15,26,33,39,50,51,57,62,68,69,75,79,82,86,93,97,99, 118,127,141,147,165,167,172,178,181,187,207,217,235, 239,242,244,248,253,257,259,271,275,277,284,293,295,...

E.g. 147 is a deficient number and $1^1 + 4^4 + 7^7 = 823800$ is an abundant number.

Are there infinitely many consecutive terms in this sequence? We conjecture: yes. Are there infinitely many *k*-tuples for these numbers?

*We remark here that with modern software freely available on the Internet, such as PARI/GP [3], it is easy to find large values of this sequence when searching a small neighborhood. For example, it took only a few seconds to find: 12345678901234567890123456793, which is a number with the property stated above.

(6) Abundant numbers such that the sum of their individual digits after being raised to their own power, is also an abundant number.

24,42,66,96,104,108,114,140,156,174,176,180,222,224,228, 270,282,288,336,352,354,392,396,400,444,448,464,516,532, 534,560,572,576,594,644,650,666,702,704,708,714,720,740,...

E.g. 24 is an abundant number and $2^2 + 4^4 = 260$ is also abundant.

Are there infinitely many consecutive numbers in this sequence?
What is the asymptotic estimate for the number of integers less than 10^m that have the property stated above?

(7) Abundant numbers such that the sum of the factorials of their individual digits is an abundant number.

36,48,54,56,66,78,84,88,96,336,348,354,364,366,368,378,
384,396,438,444,448,456,464,468,474,476,486,498,534,544,
546,558,564,576,588,594,636,644,648,654,666,678,...

E.g. 36 is an abundant number and 3!+6!= 726 is abundant.

Are there an infinite amount of odd numbers in this sequence? We conjecture: yes.
Are there an infinite amount of consecutive terms in this sequence? We conjecture: yes.

(8) Abundant-Smarandache numbers; n such that S(n) is an abundant number, where S(n) is the classic Smarandache function (A002034) [6].

243,486,512,625,972,1024,1215,1250,1536,1701,1875,1944,...

What is the 1000th term of this sequence?
Investigate this sequence.

(9) Abundant-Pseudo Smarandache numbers; n such that Z(n) is an abundant number, where Z(n) is the Pseudo-Smarandache function (A011772) [6].

13,19,25,26,31,37,39,41,42,43,49,50,56,57,61,67,70,71,
73,74,75,76,78,79,81,82,84,89,93,97,98,100,101,103,108,
109,111,113,114,121,122,127,129,133,135,139,146,147,...

Investigate these numbers.

(10) Smarandache Abundant-Partial-Digital Subsequence; the sequence of abundant numbers which can be partitioned so that each element of the partition is an abundant number. E.g. 361260 is an abundant number and it can be partitioned into 36_12_60 with 36, 12 and 60 all being abundant.

Find this sequence.

(11) Abundant numbers A such that when the smallest prime factor of A is added to the largest prime factor of A, it is also an abundant number.

5355,8415,8925,11655,13218,16065,16695,16998,19635,20778, 21105,23205,24558,25245,26436,26775,28338,29835,30555, 31815,33996,34965,37485,39654,40938,41556,42075,42735,...

E.g. the smallest prime factor of 5355 is 3 and the largest is 17; 17+3=20, an abundant number.

What are some properties of these numbers?
What are the first ten abundant numbers A, such that $A \equiv 7 \pmod{10}$?

(12) Abundant numbers A such that the sum of the composites between the smallest and largest prime factors of A is also an abundant number.

114,228,304,342,380,438,456,474,532,570,608,684,760,798,822,834,836, 876,894,906,912,948,1026,1064,1140,1182,1194,1216,1254,1314,1330,1368, 1398,1422,1460,1482,1520,1542,1580,1596,1644,1668,1672,1710,1752,1788,...

E.g. the smallest prime factor of 114 is 2 and the largest is 19. The sum of the composites between 2 and 19 is: 4+6+8+9+10+12+14+15+16+18 = 112, an abundant number.

What are some properties of these numbers? Are there any consecutive numbers in this sequence?

(13) Smarandache Nobly Abundant numbers; n such that $\tau(n)$ and $\sigma(n)$ are both abundant numbers, where $\tau(n)$ is the number of divisors of n and $\sigma(n)$ is the sum of the divisors of n.

60,84,90,96,108,126,132,140,150,156,160,180,198,204,220,224,228,234,240,252, 260, 276,294,300,306,308,315,336,340,342,348,350,352,360,364,372,380,396,414, 416,420,432,444,460,476,480,486,490,492,495,500,504,516,522,525,528,532,...

E.g. the number of divisors of 60 is 12 and the sum of the divisors of 60 is 312, both abundant numbers.

What are some properties of these numbers?

(14) Smarandache Nobly Deficient numbers; n such that $\tau(n)$ and $\sigma(n)$ are both deficient numbers, where $\tau(n)$ is the number of divisors of n and $\sigma(n)$ is the sum of the divisors of n.

1,2,3,4,7,8,9,13,16,21,25,31,36,37,43,48,49,61,64,67,73,81,93,97,100,109, 111,112,121,127,128,144,151,157,162,163,169,181,183,192,193,196,208,211, 217,219,225,229,241,256,277,283,289,313,324,331,337,361,373,381,397,400,...

Investigate this sequence.

(15) Smarandache Consecutive Abundant Digital Sum Deficient numbers; consecutive abundant numbers such that their digital sums are deficient numbers.

5984,5985
7424,7425
11024,11025
26144,26145
27404,27405
39375,39376
43064,43065
49664,49665
56924,56925
58695,58696
...

E.g. 5984 and 5985 are consecutive abundant numbers and their digital sums 5+9+8+4 = 26 and 5+9+8+5 = 27, are both deficient numbers.

Is this sequence infinite? We conjecture: yes.

(16) Smarandache Powerfully Abundant numbers. Let the abundance of n be denoted $\omega(n) = \sigma(n) - 2n$, where $\sigma(n)$ is the sum of all positive divisors of n; then the sequence is the least number m such that the abundance of m is equal to -10^n.

11,101,5090,40028,182525,2000006,

Is this sequence infinite?* What is the 100th term?

*If n is given, then it seems likely that there is some integer r>=1 such that $p = 2^r + 10^n - 1$ is prime. If it is, then $\omega(2^{r-1} * p) = -10^n$ [8].

(17) Let the deficiency of n be denoted $\alpha(n) = 2n - \sigma(n)$. Below is the sequence of n such that $\alpha(n) = \tau(n)$, where $\tau(n)$ is the number of divisors of n.

1,3,14,52,130,184,656,8648,12008,34688, 2118656,...

Is this sequence infinite? Investigate this sequence.

(18) Let the deficiency of n be denoted $\alpha(n) = 2n - \sigma(n)$. Below is the sequence of n such that $\alpha(n)$ is a perfect square* and sets a new record for such squares.

1,5,17,37,101,197,257,401,577,677,1297,1601,2597,2917,3137,4357,5477, 7057,8101,8837,12101,13457,14401,15377,15877,16901,17957,21317,22501, 24337,25601,28901,30977,32401,33857,41617,42437,44101,50177,52901,55697,...

E.g. $\alpha(37)=36$ a square which sets a new record for squares. $\alpha(101)=100$ a square which sets a new record for squares.

2597 is the only non-prime value >1 in the sequence above. What is the next non-prime value? Investigate this sequence.

*Kravitz conjectured that no numbers exist whose abundance is an odd square [9].

(19) Least deficient number of n consecutive deficients such that all are abundant numbers when they are reversed.

21,218,445,2930,4873,...

E.g. 218 is deficient and 812 is abundant, 219 is deficient and 912 is abundant; hence 218 is the least number in a chain of 2. 445 is deficient and 544 is abundant, 446 is deficient and 644 is abundant, 447 is deficient and 744 is abundant; hence 445 is the least number in a chain of 3.

Is this seqence infinite? Investigate this sequence.

(20) Let $\xi(n)$ be a function that sums the deficient numbers between the smallest and largest prime factors of n.

1,2,3,2,5,5,7,2,3,14,11,5,13,21,12,2,17,5,19,14,19,59,23,5, 5,72,3,21,29,14,31,2,57,134,12,5,37,153,70,14,41,21,43,59, 12,219,47,5,7,14,132,72,53,5,50,21,151,326,59,14,61,357,...

E.g. $\xi(n)$= 59 because the smallest and largest prime factors of 22 are 2 and 11; the sum of deficient numbers between 2 and 11 is 2+3+4+5+7+8+9+10+11 = 59. Investigate this function.

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