SOME SOLUTIONS OF THE SMARANDACHE PRIME EQUATION

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Abstract. Let k be a positive integer with k>1. In this paper we give some prime solutions $(x_1, x_2, \ldots, x_k, y)$ of the diophantine equation $y=2x_1 x_2 \ldots x_k+1$ with $2<x_1<x_2<\ldots<x_k<y$.

Let k be a positive integer with k>1. In [4, Problem 11], Smarandache conjectured that the equation

(1) $\quad y=2x_1 x_2 \ldots x_k+1, \; 2<x_1<x_2<\ldots<x_k$

has infinitely many prime solutions $(x_1, x_2, \ldots x_k, y)$ for any k. This is a very dificult problem. The equation (1) is call the Smarandache prime equation (see [3, Notion 123]), while the authors gave solutions of (1) as follows.

k=2, $(x_1, x_2, y) = (17,19,647)$;
k=3, $(x_1, x_2, x_3, y)=(3,5,19,571)$

For any positive integer n, let p_n be the n^{th} odd prime, and let $q_n = 2 p_1 p_2 \ldots p_n+1$. In this paper, by the calculating result of [1] and [2], we give nine other solutions as follows.

$$(x_1, x_2, \ldots, x_k, y) = (p_1, p_2, \ldots, p_k, q_k)$$

where k=4,10,66,138,139,311,368,495,514.

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