SURVEY ON THE RESEARCH OF SMARANDACHE NOTIONS

by M. L. Perez, editor

The American CRC Press, Boca Raton, Florida, published, in December 1998, a 2000 pages "**CRC Concise Encyclopedia of Mathematics**", by Eric W. Weisstein, ISBN 0-8493-9640-9, internationally distributed.

Among the entries included in this prestigious encyclopedia there also are the following:

- "Smarandache functions"

[i.e., Pseudosmarandache Function (p. 1459), Smarandache Ceil Function (p. 1659), Smarandache Function (p. 1660) - the most known, Smarandache-Kurepa Function (p. 1661), Smarandache Near-to-Primordial (p. 1661)], Smarandache-Wagstaff Function (p. 1663)]

- "Smarandache sequences" [41 such sequences are listed (pp. 1661-1663), in addition of 7 other Smarandache concatenated sequences (pp. 310-311))
- "Smarandache constants" [11 such constants are listed (pp. 1659-1660)]
- "Smarandache paradox" (p. 1661).

Five large pages from the above encyclopedia are dedicated to these notions.

Other contributors to the Smarandache Notions are cited as well in this wonderful mathematical treasure: C. Ashbacher, A. Begay, M. Bencze, J. Brown, E. Burton, I. Cojocaru, S. Cojocaru, J. Castillo, C. Dumitrescu, Steven Finch, E. Hamel, F. Iacobescu, H. Ibstedt, K. Kashihara, H. Marimutha, M. Mudge, I. M. Radu, J. Sandor, V. Seleacu, N. J. A. Sloane, S. Smith, Ralf W. Stephan, L. Tutescu, David W. Wilson, E. W. Weisstein, etc.

Professor Eric W. Weisstein from the University of Virginia has extended more results on Smarandache sequences, such as:

- The Smarandache Concatenated Odd Sequence:

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, ...

(Sloane's A019519) contains another prime term:

SCOS(2570) = 13579111315...51375139, which has 9725 digits!

This is the largest consecutive odd number sequence prime ever found.

Conjecture 1: There is a finite number of primes in this sequence.

- The Smarandache Concatenated Prime Sequence:

2, 23, 235, 2357, 235711, 23571113, 2357111317, ...

(Sloane's A019518) is prime for terms 1, 2, 4, 128, 174, 342, 435, 1429, ... (Sloane's A046035) with no other less than 1960.

Conjecture 2: There is a finite number of primes in this sequence.

- The Smarandache Concatenated Square Sequence:

1, 14, 149, 14916, 1491625, 149162536, 14916253649, ...

(Sloane's A019521) contains a prime only 149 (the third term) in the first 1828 terms.

Conjecture 3: There is only a prime in this sequence.

- The Smarandache Concatenated Cubic Sequence:
1, 18, 1827, 182764, 182764125, 182764125216, ...
(Sloane's A019522) contains no prime in the first 1356 terms.
Conjecture 4: There is no prime in this sequence.

David W. Wilson (wilson@cabletron.com) proved that
- The Smarandache Permutation Sequence:
12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, ...
has no perfect power in its terms.

Proof:
Their last digits should be:
either 2 for exponents of the form 4k+1,
either 8 for exponents of the form 4k+3, where $k \geq 0$.
12 is not a perfect power. All remaining elements are congruent to 2 (mod 4), and are therefore not a perfect power, either. QED.

- The Smarandache Binary Sieve (Item 29 in http;//www.gallup.unm.edu/~smarandache/SNAQINT.txt):
1,3,5,9,11,13,17,21,25,27,29,33,35,37,43,49,51,53,57,59,65,67,69, 73,75,77,81,85,89,91,97,101,107,109,113,115,117,121,123,129,131,1 33,137,139,145,149,...

(Starting to count on the natural numbers set at any step from 1:

- delete every 2-nd numbers
- delete, from the remaining ones, every 4-th numbers

... and so on: delete, from the remaining ones, every 2^k-th numbers, k = 1, 2, 3, ...)

Conjectures:

a) There are an infinity of primes that belong to this sequence;

b) There are an infinity of numbers of this sequence which are composite.

The second conjecture has been proved true by David W. Wilson: One way to see this is to note that any sequence with positive density over the positive integers contains an infinitude of composites (the density of this sequence is
1/2 * 3/4 * 7/8 * 15/16 * 31/32 * ... = 0.288788095086602421278... > 0.)
Another way to see this is to note that this sequence contains all numbers of the form $(4^k-1)/3$ for $k \geq 3$, which are all composite.

Also, in the "Bulletin of Pure and Applied Sciences", Delhi, India, Vol. 17E, No. 1, 1998 (pp. 103-114, 115-116, 117-118, 123-124) four articles present the "Smarandache noneuclidean geometries".

References:

[1] C. Dumitrescu, V. Seleacu, "Some Notions and Questions in NumberTheory",http;//www.gallup.unm.edu/~smarandache/SNAQINT.txt.
[2] E. W. Weisstein, E-mails to J. Castillo, March-December 1998.
[3] D. W. Wilson, E-mails to J. Castillo, Fall-Winter 1998.