THE 3-DIVISIBILITY OF ELEMENTS OF THE SMARANDACHE DECONSTRUCTIVE SEQUENCE

Maohua Le

Abstract . For any positive integer n, let $SDS(n)$ be the n-th element of the Smarandache deconstructive sequence . In this paper we prove that if $3^k \| n$, then $3^k \| SDS(n)$.

Key words . Smarandache deconstructive sequence , 3-divisibility .

The Smarandache deconstructive sequence is constructed by sequentially repeating the digits $1,2,\cdots,9$ in the following way :

(1) $$1,23,456,7891,\cdots,$$

which first appeared in [3] . For any positive integer n , let $SDS(n)$ denote the n-th element of this sequence . In [1] , Ashbacher showed that $3 \mid SDS(n)$ if and only if $3 \mid n$ .Simultaneously , he proposed the following question .

Question . Let k be the largest integer such that $3^k \mid n$. Is it true that

(2) $$3^k \| SDS(n) ?$$

In this paper we completely solve the mentioned question .We prove the following result.

Theorem . If $3^k \| n$, then (2) bolds.

Proof. By [1, Table1], the theorem holds for $n \leqslant 30$. Therefore, we may assume that $n > 30$. If $3^k \| n$, then

(3) $$n=3^k m,$$

where m is a positive integer with $3 \mid m$.

By the resut of [2], If $k=1$, then we have

$$(4) SDS(n)=\begin{cases} 456789.10^a+123456788889^b+123456, & \text{if } n\equiv 3 (\text{mod}9), \\ 789.10^a+123456789^b+123, & \text{if } n\equiv 6 (\text{mod}9), \end{cases}$$

where a, b are positive integers. Since $10^a \equiv 1$ (mod9) and $123456789 \equiv 0$ (mod9), we find from (4) that

$$(5) \qquad SDS(n) \equiv \begin{cases} 6 (\text{mod}9), \text{ if } n \equiv 3 (\text{mod}9), \\ 3 (\text{mod}9), \text{ if } n \equiv 6 (\text{mod}9). \end{cases}$$

Thus, by (5), we get $3 \parallel SDS(n)$. The theorem holds for $k=1$,

If $k>1$, let.

$$(6) \qquad n=9t,$$

where t is a positive integer. By (3) and (6), we get

$$(7) \qquad 3^{k-2} \parallel t.$$

Then, by the result of [2], we have

$$SDS(n)=123456789(1+10^9+\cdots+10^{9(t-1)})$$

$$(8) \qquad =123456789\left(\frac{10^{9t}-1}{10^9-1}\right).$$

Notice that $3^2 \parallel 123456789$ and $3^{k-2} \parallel (10^{9t}-1)/(10^9-1)$ by (7). We see from (8) that (2) holds. Thus, the theorem is proved.

References

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Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA