THE DIVISIBILITY OF THE SMARANDACHE COMBINATORIAL SEQUENCE OF DEGREE TWO

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Abstract: In this paper we prove that there has only the consecutive terms of the Smarandache combinatorial sequence of degree two are pairwise coprime.

Key words: Smarandache combinatorial sequences; consecutive terms; divisibility

Let r bea positive integer with $r>1$. Let $SCS(r)=\{a(r,n)\}_{n=1}^{\infty}$ be the Smarandache combinatorial sequence of degree r. Then we have $a(r,n)=n(n=1,2,\cdots,r)$ and $a(r,n)(n>r)$ is the sum of all the products of the previous terms of the sequence taking r terms at a time. In [2], Murthy asked that how many of the consecutive terms of $SCS(r)$ are pairwise coprime.

In this paper we solve this problem for $r=2$. We prove the following result.

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Theorem. For any positive integer n, we have $a(2, n+1) \equiv 0 \pmod{a(2,n)}$.

By the above mentioned theorem, we obtain the following corollary immediately.

Corollary. There has only the consecutive terms 1,2 of $SCS(2)$ are pairwise coprime.

Proof of Theorem. Let $b(n)=a(2,n)$ for any n. Then we have $b(1)=1$ and $b(2)=2$. It implies that the theorem holds for $n=1$.

By the define of $SCS(2)$, if $n>1$, then we have

$$\begin{aligned} b(n) &= b(1)b(2)+\cdots+b(n-2)b(n-1) \\ &= \frac{1}{2}\left((b(1)+\cdots+b(n-1))^2-\left(b^2(1)+\cdots+b^2(n-1)\right)\right) \end{aligned} \tag{1}$$

and

$$\begin{aligned} b(n) &= b(1)b(2)+\cdots+b(n-2)b(n-1) \\ &= \frac{1}{2}\left((b(1)+\cdots+b(n-1)+b(n))^2-\left(b^2(1)+\cdots+b^2(n-1)+b^2(n)\right)\right) \end{aligned} \tag{2}$$

using the basic properties of congruence (see [1, Chapter V]), we get from (1) and (2) that

$$\begin{aligned} b(n+1) &\equiv \frac{1}{2}\left((b(1)+\cdots+b(n-1))^2-\left(b^2(1)+\cdots+b^2(n-1)\right)\right) \\ &\equiv b(n) \equiv 0 \pmod{b(n)}. \end{aligned}$$

Thus, the theorem is proved.

References

[1] G.H. Hardy and E.M. Wright, An introduction to the theory of numbers, Oxford University Press, Oxford, 1938.

[2] A. Murthy, Some new Smarandache sequences, functions and partitions, Smarandache Notions J. 11(2000), 179-183.