THE EQUATION $S(1.2)+S(2.3)+\cdots+S(n(n+1))=S(n(n+1)(n+2)/3)$

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Abstract. For any positive integer a, let $S(a)$ be the Smarandache function of a. In this paper we prove that the title equation has only the solution n=1.

Key words: Smarandache function, diophantine equation

Let **N** be the set of all positive integers. For any positive integer a, let $S(a)$ be the Smarandache function of a. Recently, Bencze [1] proposed the following problem:

Problem Solve the equation

$$(1)\quad S(1\cdot 2)+S(2\cdot 3)+\cdots+S(n(n+1))=S\left(\frac{1}{3}n(n+1)(n+2)\right), n\in \mathbf{N}.$$

In this paper we completely solve the above-mentioned problem as follows.

Theorem The equation (1) has only the solution n=1.

Proof By the definition of the Smarandache function (see [2]), we have $S(1)=1$, $S(2)=2$ and

$$(2)\qquad S(a)\geqslant 3,\ a\geqslant 3.$$

Since $S(1.2)=S(1.2.3/3)=S(2)$, the equation (1) has a solution n=1.

Let n be a solution of (1) with $n>1$. Then, by (2), we get

$$(3)\quad S(1\cdot 2)+S(2\cdot 3)+\cdots+S(n(n+1))\geq 2+3(n-1)=3n-1.$$

Therefore, by (1) and (3), we obtain

(4) $$S\left(\frac{1}{3}n(n+1)(n+2)\right) \geq 3n-1.$$

On the other hand, since $(n+2)!=1.2\cdots n(n+1)(n+2)$, we get

(5) $$\frac{1}{3}n(n+1)(n+2) \mid (n+2)!.$$

We see from (5) that

(6) $$S\left(\frac{1}{3}n(n+1)(n+2)\right) \leq n+2.$$

The combination of (4) and (6) yields

(7) $$n+2 \geq 3n-1,$$

whence we get $n \leqslant 3/2 < 2$. Since $n \geqslant 2$, it is impossible. Thus, (1) has no solutions n with $n > 1$. The theorem is proved.

References

[1] M. Bencze, Open questions for the Smarandache function, Smarandache Notions J. 12(2001), 201-203.

[2] F. Smarandache, A function in number theory, Ann. Univ. Timisoara XVIII, 1980.

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