# THE FIRST DIGIT AND THE TRAILING DIGIT OF ELEMENTS OF THE SMARANDACHE DECONSTRUCTIVE SEQUENCE

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**Abstract**. In this paper we completely determine the first digit and the trailing digit of every term in the Smarandache deconstructive sequence.

**Key words**. Smarandache deconstructive sequence, first digit, trailing digit.

The Smarandache deconstructive sequence is constructed by sequentially repeating the digits 1,2,...,9 in the following way:

(1) $$1, 23, 456, 7891, \ldots,$$

which first appeared in [1]. For any positive integer $n$, let $SDS(n)$ be the $n$-th element of the Smarandache deconstructive sequence. Further, let $F(n)$ and $T(n)$ denote the first digit and the trailing digit of $SDS(n)$ respectively. In this paper we completely determine $F(n)$ and $T(n)$ for any positive integer $n$. We prove the following result.

**Theorem**. For any $n$, we have

(2) $$F(n)=\begin{cases} 1, & \text{if } n \equiv 0,1 \pmod 9, \\ 2, & \text{if } n \equiv 2,5,8 \pmod 9, \\ 4, & \text{if } n \equiv 3,7 \pmod 9, \\ 7, & \text{if } n \equiv 4,6 \pmod 9, \end{cases}$$

and

(3) $$T(n)=\begin{cases} 1, & \text{if } n \equiv 1,4,7 \pmod 9, \\ 3, & \text{if } n \equiv 2,6 \pmod 9, \\ 6, & \text{if } n \equiv 3,5 \pmod 9, \\ 9, & \text{if } n \equiv 0,8 \pmod 9. \end{cases}$$

**Proof.** By (1), we get

(4) $$F(n) \equiv 1+2+\cdots+(n-1)+1 \equiv \frac{n^2-n}{2}+1 \pmod 9.$$

let $a$ be a positive integer with $1 \leqslant a \leqslant 9$. we see from (4) that $F(n)=a$ if and only if $n$ is a solution of the congruence

(5) $$\frac{n^2-n}{2} \equiv a-1 \pmod 9.$$

Notice that (5) has only solutions

(6) $$n \equiv \begin{cases} 0,1 \pmod 9, & \text{if } a=1, \\ 2,5,8 \pmod 9, & \text{if } a=2, \\ 3,7 \pmod 9, & \text{if } a=4, \\ 4,6 \pmod 9, & \text{if } a=7. \end{cases}$$

Therefore, we obtain (2) by (6) immediately.

On the other hand, since

(7) $$T(n)= \begin{cases} F(n+1), & \text{if } F(n+1)>1, \\ 9, & \text{if } F(n+1)=1, \end{cases}$$

we see from (2) that (3) holds. Thus, the theorem is proved.

## Reference


[1] F. Smarandache, Only Problems, Not Soluitons, Xiquan Publishing House, Phoenix, Arizona, 1993.



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