The general term of the prime number sequence and the Smarandache prime function.

Sebastián Martín Ruiz. Avda de Regla, 43 Chipiona 11550 Cádiz Spain.

Let 's consider the function $d(i)$ = number of divisors of the positive integer number i. We have found the following expression for this function:

$$d(i) = \sum_{k=1}^{i} E\left(\frac{i}{k}\right) - E\left(\frac{i-1}{k}\right)$$

We proved this expression in the article "A functional recurrence to obtain the prime numbers using the Smarandache Prime Function".

We deduce that the folowing function:

$$G(i) = -E\left[-\frac{d(i)-2}{i}\right]$$

This function is called the Smarandache Prime Function (Reference)
It takes the next values:

$$G(i) = \begin{cases} 0 \;\; if \;\; i \;\; is \;\; prime \\ 1 \;\; if \; i \;\; is \;\; compound \end{cases}$$

Let is consider now $\pi(n)$ =number of prime numbers smaller or equal than n.
It is simple to prove that:

$$\pi(n) = \sum_{i=2}^{n} (1 - G(i))$$

Let is have too:

$$\begin{aligned} &If \;\; 1 \le k \le p_n - 1 \;\; \Rightarrow E\left(\frac{\pi(k)}{n}\right) = 0 \\ &If \;\; C_n \ge k \ge p_n \;\; \Rightarrow E\left(\frac{\pi(k)}{n}\right) = 1 \end{aligned}$$

We will see what conditions have to carry C_n.

Therefore we have te following expression for p_n n-th prime number:

$$p_n = 1 + \sum_{k=1}^{C_n} \left(1 - E\left(\frac{\pi(k)}{n}\right)\right)$$

If we obtain C_n that only depends on n, this expression will be the general term of the prime numbers sequence, since π is in function with G and G does with $d(i)$ that is expressed in function with i too. Therefore the expression only depends on n.

E[x]=The highest integer equal or less than n

Let is consider $C_n = 2(E(n \log n) + 1)$
Since $p_n \sim n \log n$ from of a certain n_0 it will be true that

$$(1) \quad p_n \leq 2(E(n \log n) + 1)$$

If n_0 it is not too big, we can prove that the inequality is true for smaller or equal values than n_0.

It is necessary to that:

$$E\left[\frac{\pi(2(E(n \log n)+1))}{n}\right] = 1$$

If we check the inequality:

$$(2) \quad \pi(2(E(n \log n) + 1)) < 2n$$

We will obtain that:

$$\frac{\pi(C_n)}{n} < 2 \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] \leq 1 \quad ; \quad C_n \geq p_n \Rightarrow E\left[\frac{\pi(C_n)}{n}\right] = 1$$

We can experimentaly check this last inequality saying that it checks for a lot of values and the difference tends to increase, wich makes to think that it is true for all n.

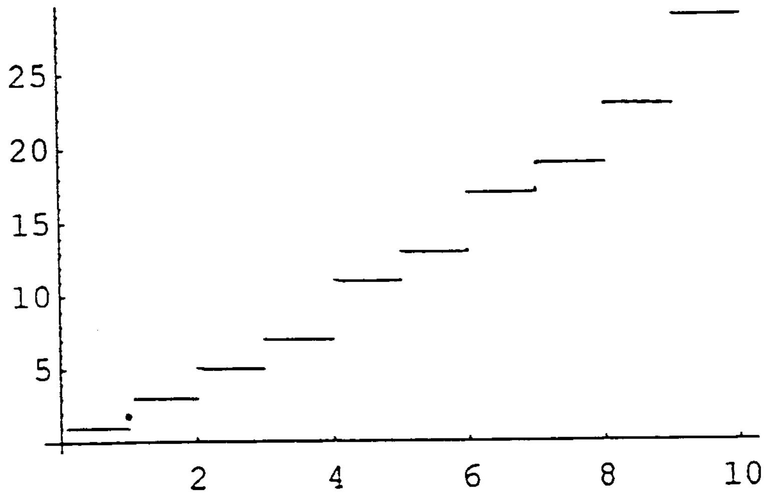
Therefore if we prove that the next inequalities are true:

$$(1) \quad p_n \leq 2(E(n \log n) + 1)$$
$$(2) \quad \pi(2(E(n \log n) + 1)) < 2n$$

which seems to be very probable; we will have that the general term of the prime numbers sequence is:

$$p_n = 1 + \sum_{k=1}^{2(E(n \log n)+1)} \left[1 - E\left[\frac{\sum_{j=2}^{k} \left[1 + E\left[-\frac{\sum_{s=1}^{j} (E(j/s) - E((j-1)/s)) - 2}{j} \right] \right]}{n} \right] \right]$$

If now we consider the general term defined in the same way but for all real number greater than zero the following grafic is obtained:



Let is observe that if $0<x<1$ P(x)=1 si x=1 P(x)=2 and for $n-1<x\leq n$ P(x)= p_n

Reference:

[1] E. Burton, "Smarandache Prime and Coprime Functions" Http://www.gallup.unm.edu/~Smarandache/primfnct.txt

[2] F. Smarandache, "Collected Papers", Vol. II, 200 p.,p.137, Kishinev University Press.