

THE LIMIT OF THE SMARANDACHE DIVISOR SEQUENCES

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Abstract. In this paper we prove that the limit $T(n)$ of the Smarandache divisor sequence exists if and only if n is odd.

Key words. Smarandache divisor sequence, limit, existence.

For any positive integers n and x , let the set
 (1) $A(n) = \{x \mid d(x) = n\}$,
 where $d(x)$ is the number of distinct divisors of x .
 Further, let

$$(2) \quad T(n) = \sum_x \frac{1}{x},$$

where the summation sign \sum denote the sum through over all elements x of $A(n)$. In [2], Murthy showed that $T(n)$ exists if $n=1$ or n is an odd prime, but $T(2)$ does not exist. Simultaneous, Murthy asked that whether $T(n)$ exist for $n=4,6$ etc. In this paper we completely solve the mentioned problem. We prove a general result as follows.

Theorem. $T(n)$ exists if and only if n is odd.

Proof. For any positive integer a with $a > 1$, let

$$(3) \quad a = p_1^{r_1} p_2^{r_2} \dots p_k^{r_k}$$

be the factorization of a . By [1, Theorem 27], we have

$$(4) \quad d(a) = (r_1 + 1)(r_2 + 1) \dots (r_k + 1).$$

If n is even, then from (1) and (4) we see that

$A(n)$ contains all positive integers x with the form

$$(5) \quad x = pq^{n/2-1},$$

where p, q are distinct primes. Therefore, we get from (2) and (5) that

$$(6) \quad T(n) > \frac{1}{2^{n/2-1}} \sum^* \frac{1}{p} = \frac{1}{2^{n/2-1}} [T(2) - 2],$$

where the summation sign \sum^* denote the sum through over all odd primes p . Since $T(2)$ does not exist, we find from (6) that $T(n)$ does not exist if n is even.

Let

$$(7) \quad n = d_1 d_2 \cdots d_t$$

be a multiplicative partition of n , where d_1, d_2, \dots, d_t are divisors of n with $1 < d_1 \leq d_2 \leq \dots \leq d_t$. Further, let

$$(8) \quad T(d_1 d_2 \cdots d_t) = \{x \mid x = p_1^{d_1-1} p_2^{d_2-1} \cdots p_t^{d_t-1}, p_1, p_2, \dots, p_t \text{ are distinct primes}\}.$$

By (1), (4), (7) and (8), we get

$$(9) \quad T(n) = \sum^{**} T(d_1 d_2 \cdots d_t),$$

where the summation sign \sum^{**} denote the sum through over all distinct multiplicative partitions of n . For any positive integer m , let

$$(10) \quad R(m) = \sum_{k=1}^{\infty} \frac{1}{k^m}$$

be the Riemann function. If n is odd, then from (7) we see that $d_1 \geq 3$. Therefore, by (4), (8), (9) and (10), we obtain

$$(11) \quad \begin{aligned} T(n) &< \sum^{**} \left[\prod_{i=1}^t \left[\frac{1}{2^{d_i-1}} \sum^* \frac{1}{p^{d_i-1}} \right] \right] \\ &< \sum^{**} \left[\prod_{i=1}^t R(d_i-1) \right] \leq \sum^{**} \left[\prod_{i=1}^t R(2) \right] \\ &= \sum^{**} [R(2)]^t. \end{aligned}$$

Since the number of multiplicative partitions of n is finite and $R(2) = \pi^2/6$, we see from (11) that $T(n)$ exists if n is odd. Thus, the theorem is proved.

References

- [1] G.H. Hardy and E.M. Wright, An Introduction to the Theory of Numbers, Oxford University press, Oxford, 1937.
- [2] A.Murthy, Some new Smarandache sequences, functions and partitions, Smarandache Notions J. 11(2000), 179-185.

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