THE LOWER BOUND FOR THE SMARANDACHE COUNTER C(0,n!)

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Abstract. In this paper we prove that if n is an integer with $n \geq 5$, then the Smarandache counter C(0,n!) satisfies $C(0,n!) > (1-5^{-k})(n+1)/4-k$, where $k=\lfloor \log n / \log 5 \rfloor$.

Let a be an integer with $0 \leq a \leq 9$. For any positive decimal integer m, the number of a in digits of m is called the Smarandache counter of m with a. It is denoted by C(a,m) (see [1,Notion 132]). Let n be a positive integer. In this paper we give a lower bound for C(0,n!) as follows:

Theorem. If $n \geq 5$, then we have

(1) $C(0,n!) > 1/4(1-5^{-k})(n+1)-k$,

where $k=\lfloor \log n / \log 5 \rfloor$

Proof. Let

(2) $n! = \overline{a_s a_{s-1} \ldots a_1 a_0}$.

If $n \geq 5$, then we have $10 | n!$ and $a_0=0$. Further, let $2^u \| n!$ and $5^v \| n!$. By [2, Theorem 1*11*1], we get

(3) $u=\sum_{r=1}^{\infty}[n/2^r]$, $v=\sum_{r=1}^{\infty}[n/5^r]$.

We see from (3) that $u \geq v$. It implies that there exist continuous v zeros $a_0=a_1=\ldots=a_{v-1}=0$ in (2). So we have

(4) $C(0,n!) \geq v$.

Let $k=[\log n/\log 5]$. Since $[n/5^r]=0$ if $r>k$, we see from (3) that

(5) $v=\sum_{r=1}^{\infty}[n/5^r]$

Since $[n/5^r] \geq n/5^r-(5^r-1)/5^r$, we get from (5) that

(6) $v \geq \sum_{r=1}^{k}(n/5^r-(5^r-1)/5^r)=1/4(1-5^{-k})(n+1)-k$

Substitute (6) into (4) yelds (1). The theorem is proved.

Reference

1. Dumitrescu and Seleacu, Some Notions and Questions In Number Theory, Erhus Univ. Press, Glendale, 1994.
2. L.-K.Hua, Introduction to Number Theory, Springer, Berlin, 1982.