THE MODULE PERIODICITY OF SMARANDACHE CONCATENATED ODD SEQUENCE

Xigeng Chen
Maoming Educational College, Maoming, Guangdong, P.R.China

Maohua Le
Zhanjiang Normal College, Zhanjiang, Guangdong, P.R.China

Abstract

In this paper we prove that the residue sequence of Smarandache concatenated odd sequence mod 3 is periodical.

Let p be a prime. For any integer a, let $<a>_p$ denote the least nonnegative residue of a mod p. Furter, for an integer sequence

$A = \{a(n)\}_{n=1}^{\infty}$, the sequence $\{<a(n)>_p\}_{n=1}^{\infty}$ is called the residue sequence of A mod p, and denoted by $<A>_p$.

In [1], Marimutha defined the Smarandache concatenated odd

sequence $S=\{s(n)\}_{n=1}^{\infty}$, where

(1) $s(1)=1,\ \ s(2)=13,\ \ s(3)=135\ ,\ \ s(4)=1357, \ldots$.

In this paper we discuss the periodicity of $<S>_p$. Clearly, if p=2 or 5, then the residue sequence $<S>_p$ is periodical. We now prove the following result:

Theorem. If p=3, then $<S>_p$ is periodical.

Prof. For ahy positive integer k, we have $10^k \equiv 1 \pmod 3$.

Hence, we see from (1) that

(2) $s(n) \equiv 1+3+5+\ldots+(2n-1) = n^2 \pmod 3$.

Since

(3) $<n^2>_3 = \begin{cases} 0, \text{ if } n \equiv 0 \pmod 3; \\ 1, \text{ if } n \equiv 1 \text{ or } 2 \pmod 3, \end{cases}$

we find from (2) and (3) that

(4) $$\langle s(n)\rangle_3 = \begin{cases} 0, & \text{if } n \equiv 0 \pmod 3; \\ 1, & \text{if } n \equiv 1 \text{ or } 2 \pmod 3, \end{cases}$$

Thus, by (4), the sequence $\langle S\rangle_3 = \{\langle s(n)_3\rangle\}_{n=1}^{\infty}$ is periodical.
The theorem is proved.

Finally, we pose the following

Question. Is the residue sequence $\langle S\rangle_p$ periodical for every odd prime p?

Reference:

1.H.Marimutha, "Smarandache concatenate type sequences", Bulletin of Pure and Applied Sciences, 16E (1997), No. 2, 225 - 226.